I need two things for a numerical evaluation: c_n, $\frac{\partial G}{\partial \eta}$

c_n: $c_n = a_n b_n$

$$b_n = \int_0^1 \Phi_n (1-\eta^2)\,\eta\, d\eta \qquad \Phi_n = a_n G(\lambda_n \eta)$$

$\langle \Phi_n, \Phi_n \rangle = 1$ normalization condition to find a_n

$$\int_0^1 \Phi_n \Phi_n (1-\eta^2)\,\eta\, d\eta = 1$$

$$\int_0^1 a_n^2 G_n^2(\lambda_n \eta)(1-\eta^2)\,\eta\, d\eta = 1$$

$$a_n^2 = \left[\int_0^1 G_n^2(\lambda_n \eta)(1-\eta^2)\,\eta\, d\eta\right]^{-1}$$

$$c_n = a_n b_n = a_n \int_0^1 a_n G_n(\lambda_n \eta)(1-\eta^2)\, d\eta$$

$$= a_n^2 \int_0^1 G_n(\lambda_n \eta)(1-\eta^2)\, d\eta$$

$$c_n = \frac{\int_0^1 G_n(\lambda_n \eta)(1-\eta^2)\,\eta\, d\eta}{\int_0^1 G_n^2(\lambda_n \eta)(1-\eta^2)\,\eta\, d\eta}$$

$\frac{\partial G}{\partial \eta}$:

$$G(\eta) = e^{-\lambda \eta^2/2}\, M\left(\tfrac{1}{2} - \tfrac{\lambda}{4}, 1, \lambda \eta^2\right) \qquad a = \tfrac{1}{2} - \tfrac{\lambda}{4}, \quad b = 1$$

Need derivative of $M(a,b,z)$:

$$\frac{\partial M}{\partial z} = \frac{a}{b} M(a+1, b+1, z)$$

From NIST

https://dlmf.nist.gov/13.3

$$\frac{\partial G}{\partial \eta} = e^{-\lambda\eta^2/2} \cdot -\frac{\lambda}{2} \cdot 2\eta \cdot M(a,b,\lambda\eta^2) + e^{-\lambda\eta^2/2} \cdot \frac{\partial M}{\partial \eta}$$

$$\frac{\partial M}{\partial \eta} = \frac{\partial M}{\partial z} \cdot \frac{\partial z}{\partial \eta} = \frac{\partial M}{\partial z} \cdot 2\lambda\eta \qquad z = \lambda\eta^2$$

$$\frac{\partial M}{\partial z} = \frac{a}{b} M(a+1, b+1, z)$$

$$\frac{\partial G}{\partial \eta} = -\lambda\eta e^{-\lambda\eta^2/2} M(a,b,\lambda\eta^2) + 2\lambda\eta e^{-\lambda\eta^2/2} \cdot \frac{a}{b} M(a+1, b+1, \lambda\eta^2)$$

$$= \lambda\eta e^{-\lambda\eta^2/2} \left[-M(a,b,\lambda\eta^2) + 2\frac{a}{b} M(a+1, b+1, \lambda\eta^2) \right]$$

$$= \lambda\eta e^{-\lambda\eta^2/2} \left[-M\left(\tfrac{1}{2} - \tfrac{\lambda}{4}, 1, \lambda\eta^2\right) + \frac{2\left(\frac{1}{2} - \frac{\lambda}{4}\right)}{1} \cdot M\left(\tfrac{3}{2} - \tfrac{\lambda}{4}, 2, \lambda\eta^2\right) \right]$$

$$\frac{\partial G}{\partial \eta} = \lambda\eta e^{-\lambda\eta^2/2} \left[\left(1 - \tfrac{\lambda}{2}\right) M\left(\tfrac{3}{2} - \tfrac{\lambda}{4}, 2, \lambda\eta^2\right) - M\left(\tfrac{1}{2} - \tfrac{\lambda}{4}, 1, \lambda\eta^2\right) \right]$$