Special Problem A-3

In this problem, we are going to derive important relationships for spherical coordinates. Note that all of the answers to this problem can be found in appendix A of your textbook, so you must carefully show your work in order to get credit.

(a) Starting from the position vector $\boldsymbol{r} = x\boldsymbol{e_x} + y\boldsymbol{e_y} + z\boldsymbol{e_z}$ and the mapping between $\{x, y, z\}$ and $\{r, \theta, \phi\}$, determine the scale factors $\{h_r, h_\theta, h_\phi\}$.

(b) Determine the base vectors $\{\boldsymbol{e_r}, \boldsymbol{e_\theta}, \boldsymbol{e_\phi}\}$ for spherical coordinates in terms of the Cartesian base vectors.

(c) Write your answer from (b) as a matrix, $\boldsymbol{Q}$ where

$$\begin{bmatrix} \boldsymbol{e_r} \\ \boldsymbol{e_\theta} \\ \boldsymbol{e_\phi} \end{bmatrix} = \boldsymbol{Q} \cdot \begin{bmatrix} \boldsymbol{e_x} \\ \boldsymbol{e_y} \\ \boldsymbol{e_z} \end{bmatrix}$$

Verify that

$$\begin{bmatrix} \boldsymbol{e_x} \\ \boldsymbol{e_y} \\ \boldsymbol{e_z} \end{bmatrix} = \boldsymbol{Q}^t \cdot \begin{bmatrix} \boldsymbol{e_r} \\ \boldsymbol{e_\theta} \\ \boldsymbol{e_\phi} \end{bmatrix}$$

Hint: There are different ways to do this, but one is to show that $\boldsymbol{Q} \cdot \boldsymbol{Q}^t = \boldsymbol{\delta}$ *(the identity tensor).*

(d) Compute the partial derivatives $\{\partial/\partial r, \partial/\partial\theta, \partial/\partial\phi\}$ of each of the base vectors. Express these nine derivatives in terms of the spherical base vectors. In other words, derive Eq. A.7-41 in your textbook.