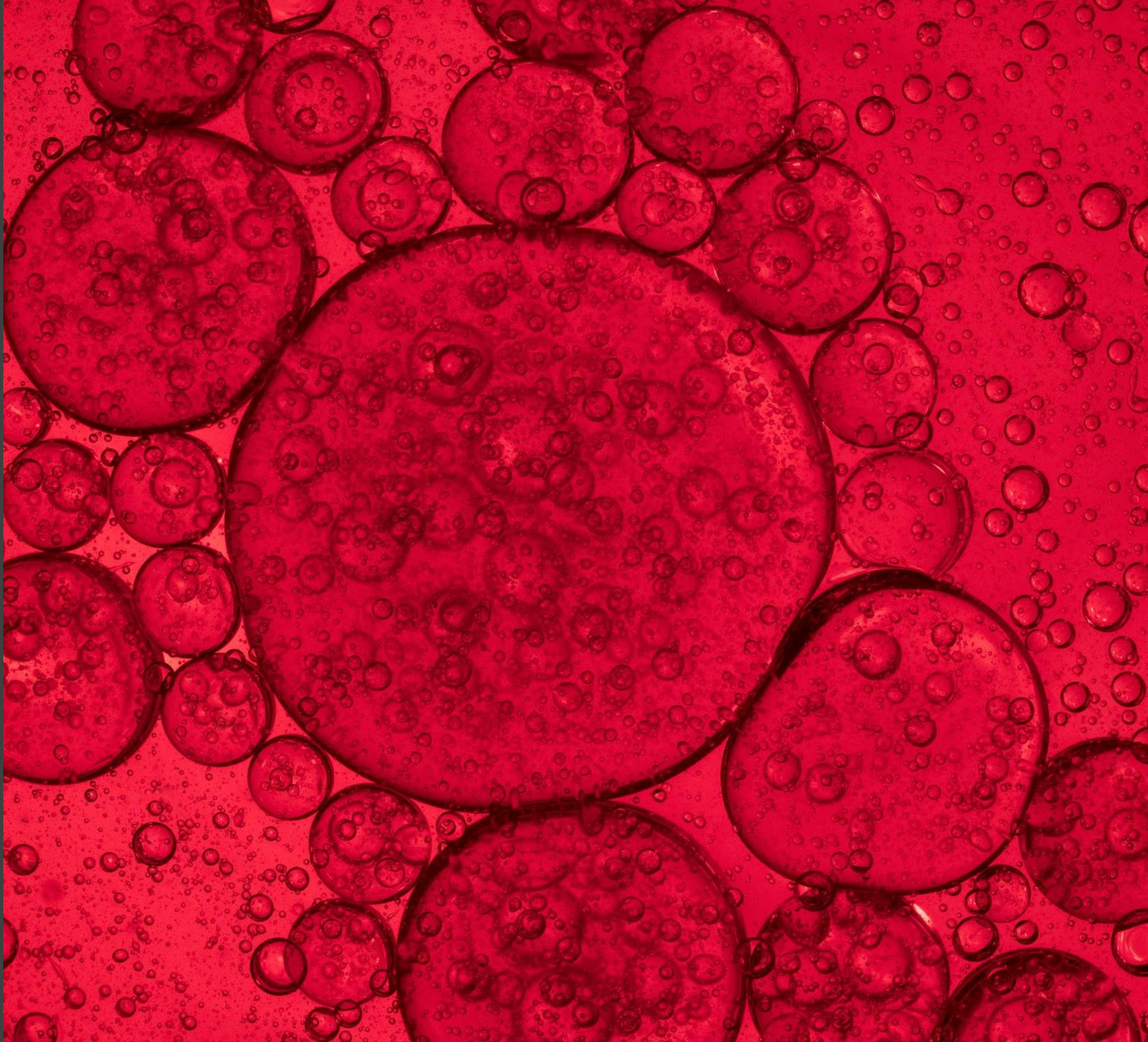
DETERMINING THE VELOCITY PROFILE OF BLOOD

ChEn 533
12/14/2020



Rheology of Blood

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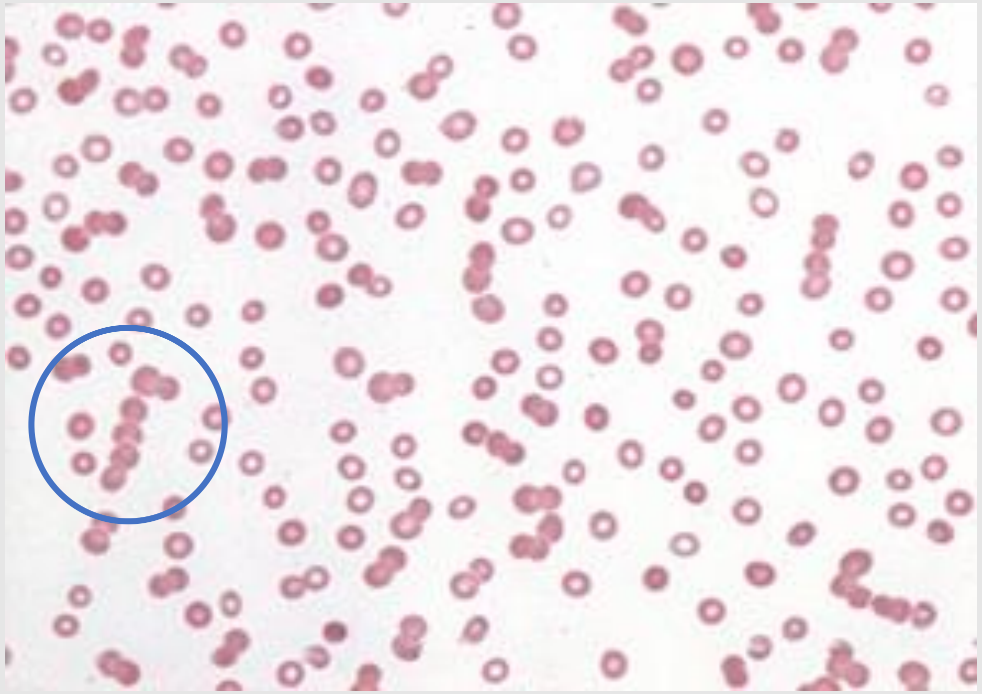
Why blood?

- Blood is a **multi-phase** mixture
- The solid red blood cells are elastic and cupped
- This leads to **rouleaux** clumps
- These properties make blood **non-Newtonian**

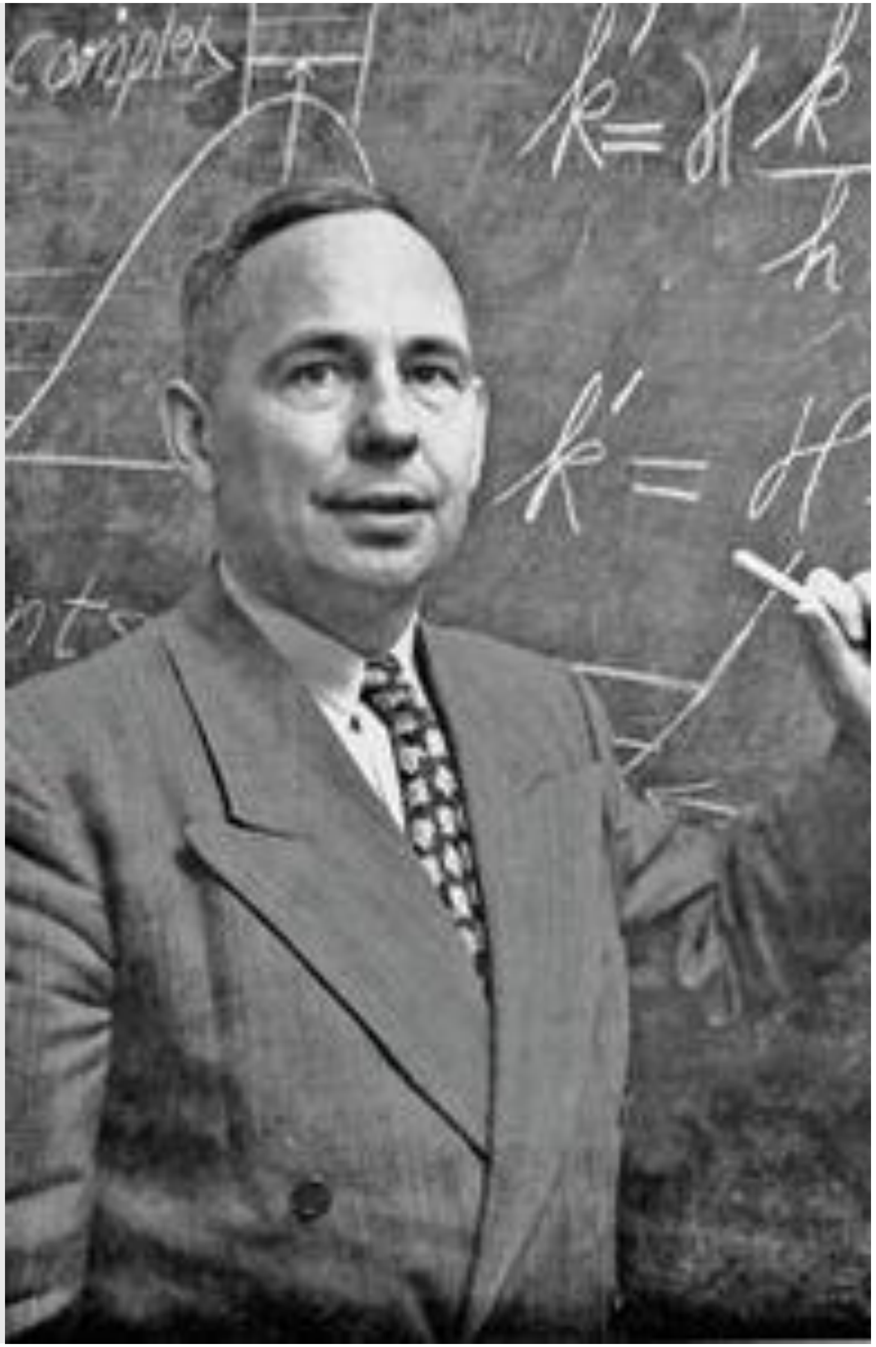


Why do we want to know this profile?

- Improved treatment and diagnosis methods
- Anemia
- Afibrinogenemia
- Blot Clots

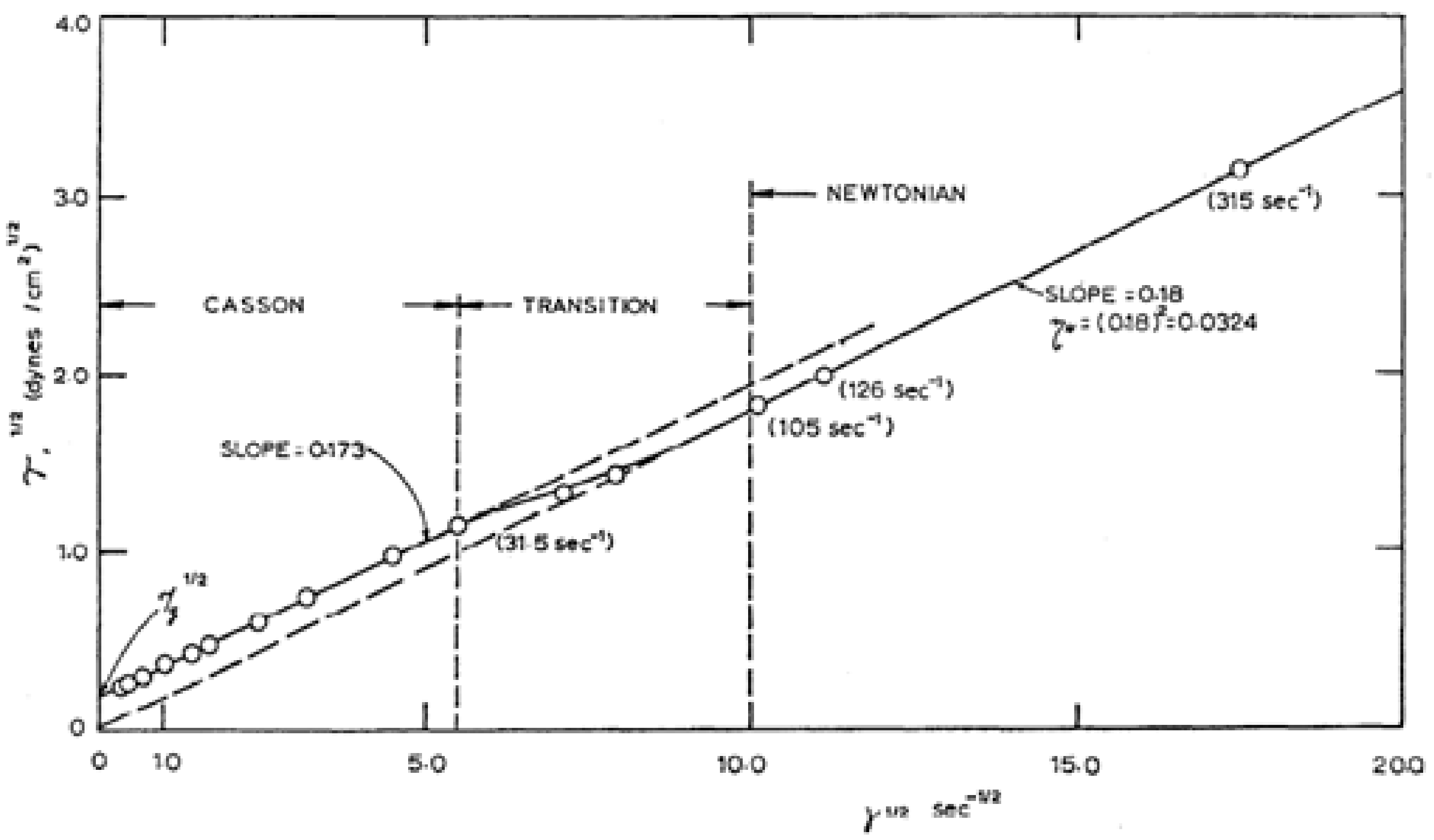
Non-Newtonian Fluids Require a Model

- Casson model
- Other models exist



IDEA BEHIND THE CASSON MODEL

$$\tau_{rz}^{1/2} = \tau_y^{1/2} + s\dot{\gamma}^{1/2}$$



How to use this to determine velocity?

- Write the Casson equation in terms of the shear rate
- Perform a force balance on a control volume
- Integrate

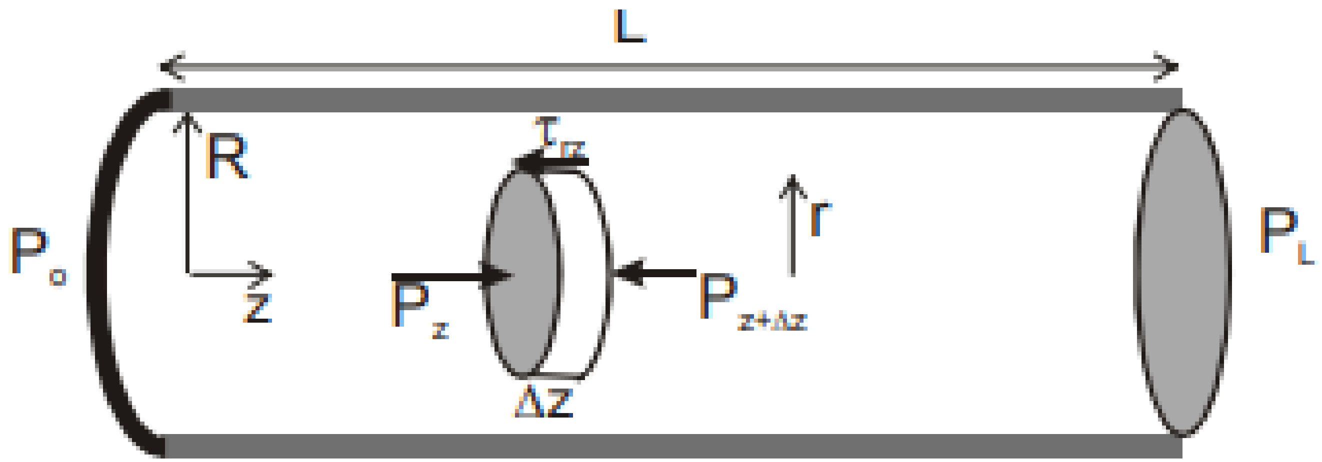
Assumptions

- No slip
- Steady
- Fully-developed
- Constant density
- No entrance effects ($L/R > 100$)
- Isothermal
- One-direction momentum transfer
- Typical hematocrit levels

STEP 1

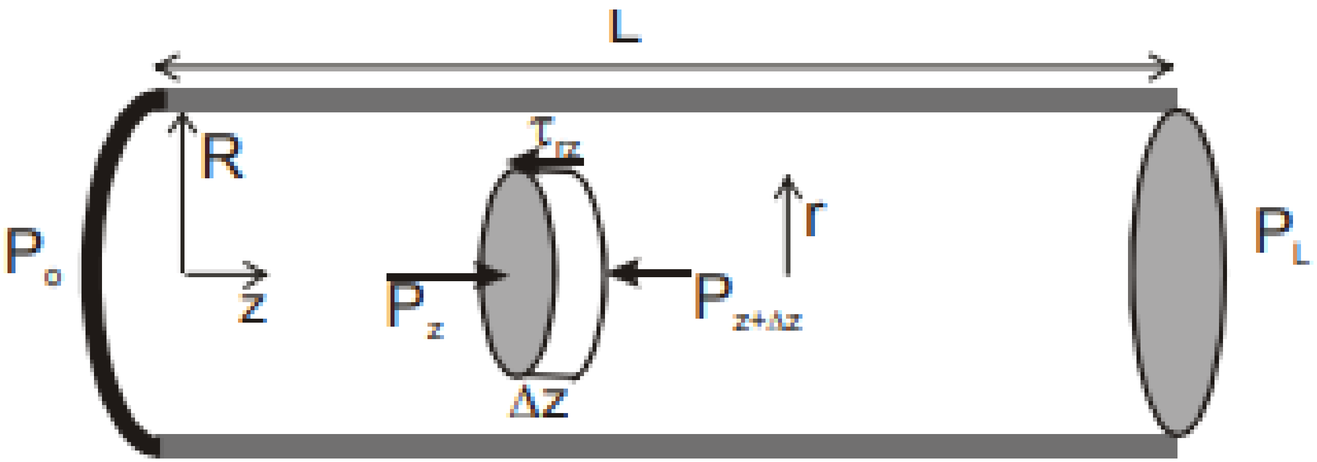
$$\tau_{rz}^{1/2} = \tau_y^{1/2} + s\dot{\gamma}^{1/2}$$

$$\dot{\gamma} = -\frac{dv_z}{dr} = \frac{1}{s^2}\left(\tau_{rz}^{1/2} - \tau_y^{1/2}\right)^2$$



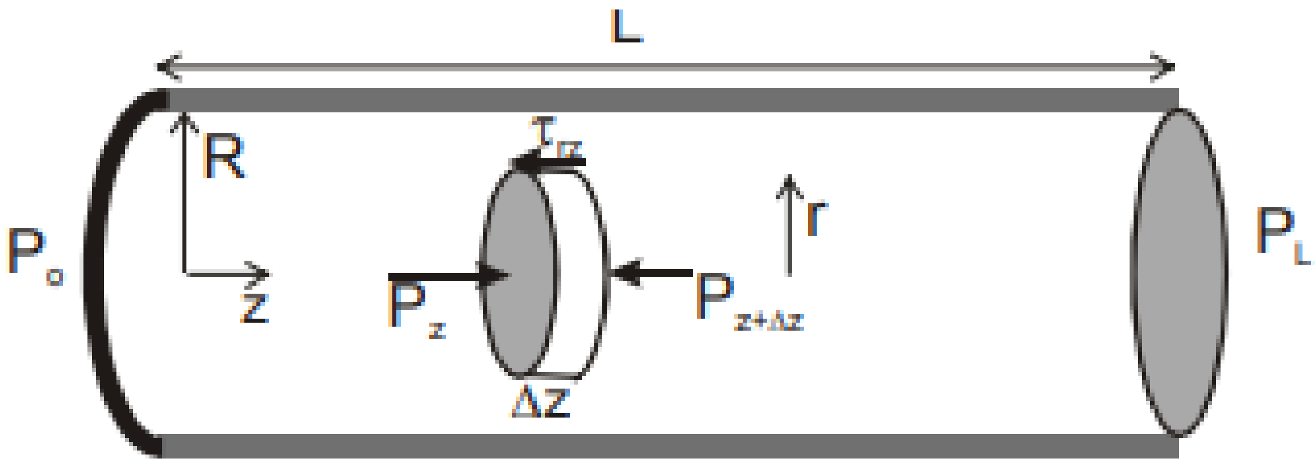
STEP 2

$$\pi r^2 P|_z - \pi r^2 P|_{z+\Delta z} - 2\pi r \Delta z \tau_{rz} = 0$$



STEP 2

$$\frac{2}{r}\tau_{rz} = -\frac{dP}{dz}$$



STEP 2

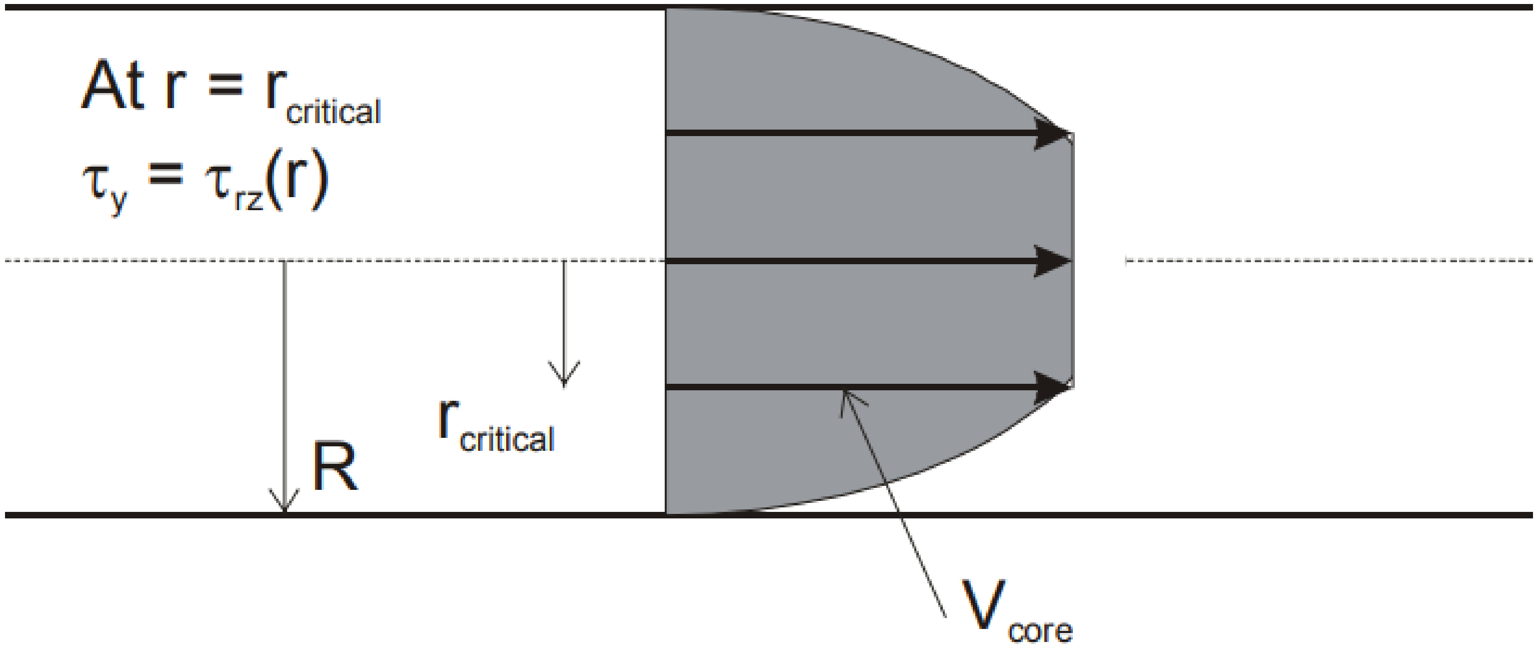
$$\tau_{rz} = -\frac{(P_O - P_L)}{2L} r$$

$$\dot{\gamma} = -\frac{dv_z}{dr} = \frac{1}{s^2}\left[\frac{\tau_w}{R}r - 2\left(\frac{\tau_w}{R}\right)^{1/2} r^{1/2} \tau_y^{1/2} + \tau_y\right]$$

STEPS 1 & 2

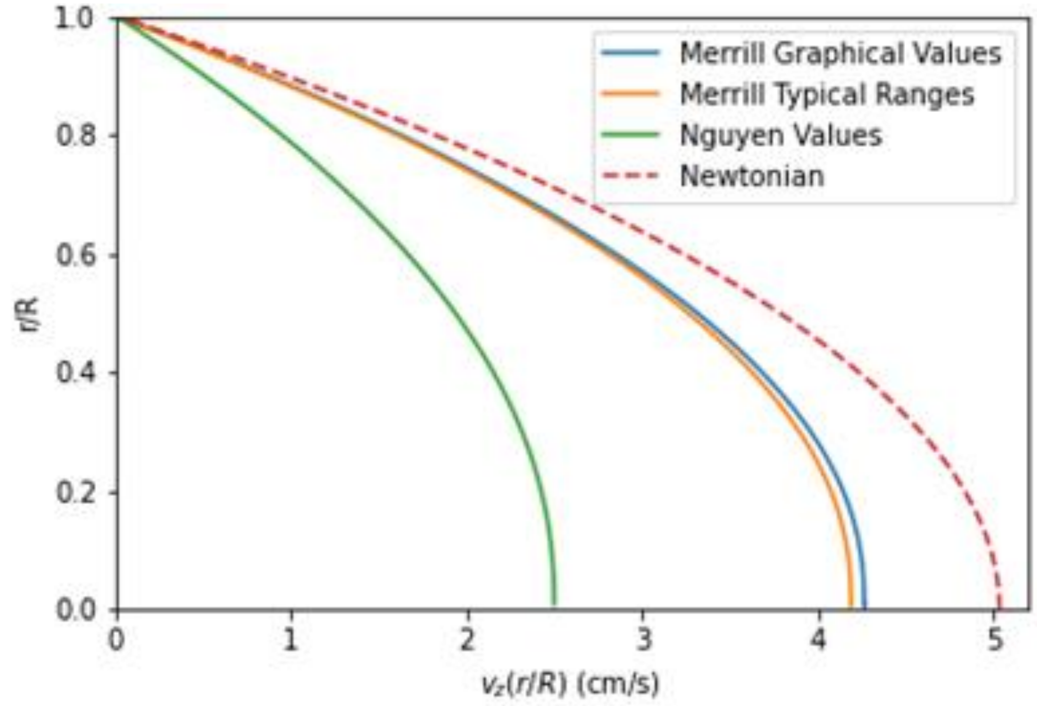
$$v_z(r) = \frac{R\tau_w}{2s^2}\left\{\left[1 - \left(\frac{r}{R}\right)^2\right] - \frac{8}{3}\left(\frac{\tau_y}{\tau_w}\right)^{1/2}\left[1 - \left(\frac{r}{R}\right)^{3/2}\right] + 2\left(\frac{\tau_y}{\tau_w}\right)\left(1 - \frac{r}{R}\right)\right\}$$

STEP 3: INTEGRATE

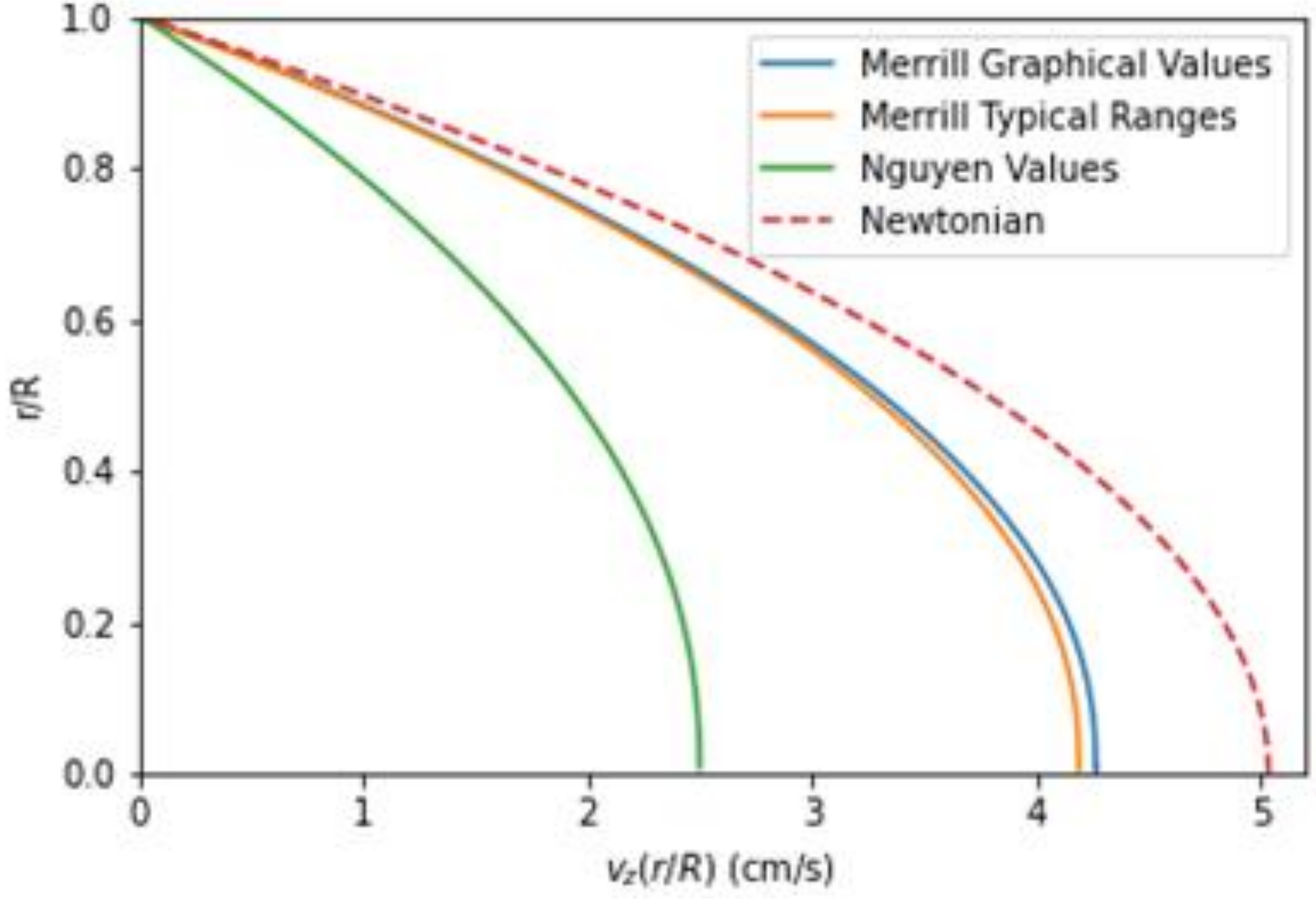


YIELD STRESS OPPOSES MOTION

RESULTS



1.0
0.8
0.6
0.4
0.2
0.0
r/R
0
1
2
3
4
5
$v_z(r/R)$ (cm/s)
Merrill Graphical Values
Merrill Typical Ranges
Nguyen Values
Newtonian

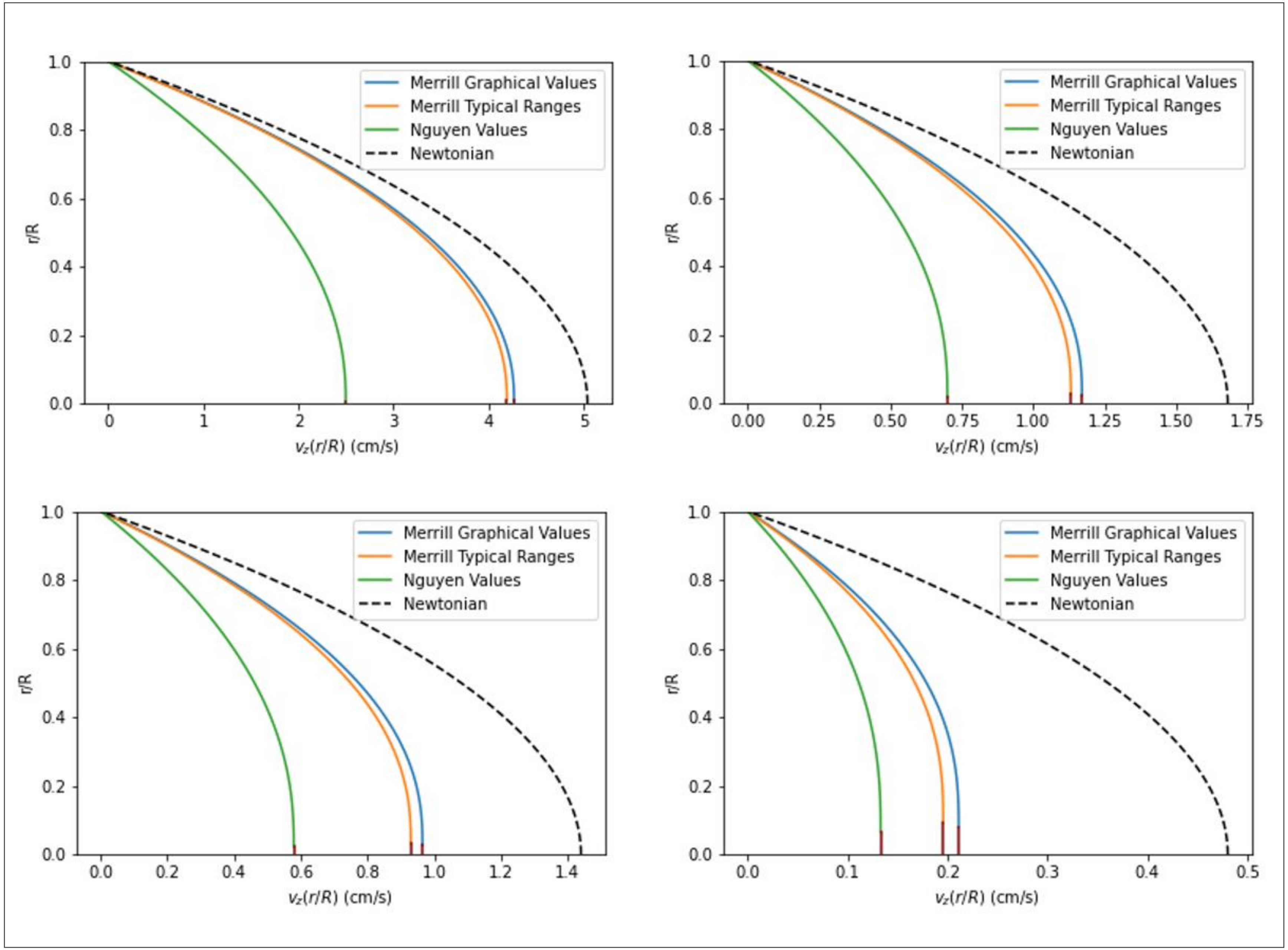


Results

- Blunter profile than Newtonian flow
- Tiny critical radius
- Dependent on pressure and length

Pressure and Length Dependence

- Top Left: Typical
- Top Right: Longer Length
- Bottom Left: Lower Pressure
- Bottom Right: Longer Length and Lower Pressure



Impact

PAVED WAY TO FURTHER UNDERSTANDING OF BLOOD

ABILITY TO DIAGNOSE AND TREAT DISEASES

FURTHER IMPLICATIONS INTO PHARMACEUTICALS AND FOODS

QUESTIONS?

Sources

- 1. Merrill, Edward W. “Rheology of Blood.” *Physiologial Reviews*, vol. 49, no. 4, Oct. 1969, pp. 863–884.
- 2. Nguyen, Thaun K. “The Physical and Flow Properties of Blood.” *Chapter 3*, Cal Poly Pomona, www.cpp.edu/~tknguyen/che499/Notes/Chapter3.pdf.
- 3. Raza, Jawad. “Multiple Solutions of Mixed Convective MHD Casson Fluid Flow in a Channel.” *Journal of Applied Mathematics*, 4 Sept. 2016, doi:10.1155/2016/7535793. 4. Sochi, Taha. “Non-Newtonian Rheology in Blood Circulation.” *ArXiv - University College London*, 9 June 2014, pp. 1–26., arxiv.org/pdf/1306.2067.pdf#:~:text=Blood%20is%20a%20complex%20non%2DNewtonian%20fluid%20and%20hence%20reliable,non%2DNewtonian%20character%2D%20istics
- Images from:
- https://en.wikipedia.org/wiki/Jean_L%C3%A9onard_Marie_Poiseuille (slide 3)
- https://en.wikipedia.org/wiki/Henry_Eyring_(chemist) (slide 4)

Model	τ_y	s
Merrill Graphical Values	0.035	$0.03^{1/2}$
Merrill Typical Ranges	0.04	$0.03^{1/2}$
Nguyen Values	0.0289	0.229
Newtonian	0	$0.0324^{1/2}$

APPENDIX