Example.

A power-law fluid has $m = 0.01 \frac{kg}{m s^{1.25}}$, $n = 0.75$ and $\rho = 10^3\ kg/m^3$

It is flowing in a pipe of $D = 3cm$ at a flow rate of $Q = 2\,L/s$. What is Re_{PL}?

$$Re_{PL} = \frac{8}{\left[\frac{2(3n+1)}{n}\right]^n} \frac{\rho u^{2-n} D^n}{m}$$

$$u = \frac{4Q}{\pi D^2} = 2.829\ m/s$$

$$Re_{PL} = \frac{8}{\left[\frac{2(3 \times 0.75+1)}{0.75}\right]^{0.75}} \frac{10^3\ \cancel{kg/m^3}\ (2.829)^{1.25} \cancel{\frac{m^{1.25}}{s^{1.25}}}\ (0.03)^{0.75}\ \cancel{m^{0.75}}}{0.01\ \cancel{\frac{kg}{m\, s^{1.25}}}}$$

$$\boxed{Re_{PL} = 4.19 \times 10^4}$$

<u>Example</u> : For the fluid in the previous example, calculate the pressure drop over 50 meters of pipe that increases 4 meters in elevation.

$D = 3cm$

$\rho = 10^3 \; kg/m^3$

$u = 2.829 \; m/s$

$m = 0.01 \; kg/ms^{1.25}$

$n = 0.75$

* calculate $f = f(Re_{PL})$ first.

$$\frac{1}{\sqrt{f}} = \frac{4.0}{n^{0.75}} \log\left(Re_{PL} \, f^{\frac{2-n}{2}} \right) - \frac{0.4}{n^{1.2}}$$

↳ implicit → use <u>fsolve</u>.

$f = 0.0044$

* calculate dynamic pressure

$$\Delta \mathcal{P} = \frac{-2\rho u^2 f L}{D}$$

$$= -117.5 \text{ kPa}$$

* calculate pressure drop

$$\Delta \mathcal{P} = \Delta P + \rho g \Delta h$$

$$\Delta P = \Delta \mathcal{P} - \rho g \Delta h$$

$$= -117.5 \text{ kpa} - \left(10^3 \frac{kg}{m^3}\right)\left(9.8 \, m/s^2\right)(4m)$$

$$\boxed{\Delta P = -156.7 \text{ kPa}}$$

In [12]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve
```

In [13]:
```
# Define our givens

m = 0.01 # kg m^-1 s^-1.25
n = 0.75
rho = 1e3 # kg m^-3

Q = 2e-3 # m^3/s
D = 0.03 # m
```

In [14]:
```
# calculate U and Re
U = 4*Q/np.pi/D**2 # m/s
Re = 8/(2*(3*n+1)/n)**n * rho*U**(2-n)*D**n/m

print("U = %f m/s"%U)
print("Re = %1.4e"%Re)
```

```
U = 2.829421 m/s
Re = 4.1895e+04
```

In [15]:
```
# Define Dodge-Metzner correlation in standard form
def g(f):
    return 4/n**0.75 * np.log10(Re*f**(1-n/2.))-0.4/n**1.2 - f**(-0.5)
```

In [17]:
```
# solve for f
f = fsolve(g, 0.001)

print('f = %f'%f)
```

```
f = 0.004405
```

In [28]:
```
# Solve for dynamic pressure
L = 50 # m

D_Pdyn = -2*rho*U**2*f*L/D # Pa

print('D_Pdyn = %0.3f kPa'%(D_Pdyn/1.e3))
```

```
D_Pdyn = -117.544 kPa
```

In [33]:
```
# Solve for pressure drop
g = 9.8 #m/s^2
D_h = 4 # m

D_P = D_Pdyn - rho*g*D_h

print('D_P = %0.3f kPa'%(D_P/1.e3))
```

```
D_P = -156.744 kPa
```