Lecture 28 – Combined Engineering Balances

* Reading: Review Deen ch. 11

* Thought, Prayer, Quiz

I. Engineering Design

* Now that we know all of the integral balances, we are ready to start thinking about design.

* The purpose of an engineering design is to solve a practical problem. By nature such problems are inherently open-ended.

* Because they are open-ended, it is useful to have a process we can apply to help us think about how to do design. The process we are going to use is the "ASE" process.

A – Analysis

S – Synthesis

E – Evaluation

Analysis: In this stage you are defining the problem.

This is not always a simple matter. You are setting goals here. What should your goals be? Are they feasible? (e.g. the iPad).

Synthesis: This stage has two parts:

(1). Generate possible solutions

(2). Pick one & do it.

Part (1) takes creativity to come up with ideas of how to achieve your goal. Part (2) takes persistence & technical ability to do it.

Evaluation: In the last stage, we step back and assess our solution relative to our goals. There are important technical & non-technical factors here. For example, does your design meet gov't regulations? Is it safe? How will it impact the environment or society?

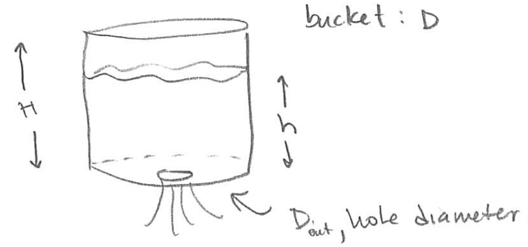
* Today we are going to start using "ASE" to solve engineering design problems using multiple integral balances.

* I am providing you with a handout that reviews the integral balances. Hopefully this handout & the extra practice will help you learn the integral balance material better as well.

II. Example problems

A. mass flow rate of a draining bucket.

* Suppose I have a bucket with a hole in it. Design a method to measure the mass flow rate out of the hole.



* Design: Analysis ✓

  I did for you. why might we want to know this?

Synthesis: (1) Generate Possible solutions: (students do)

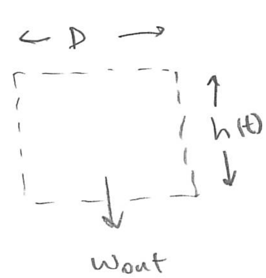
- timer & a ruler
- differential balance
- integral balances.

(2) Pick one. → I am going to use integral balances. (I do)

- why not use a differential balance? Hard, Don't need flow details.
- what type of integral balances?
  - mass for sure, maybe Bernoulli, b/c I will need a velocity at the bottom.

mass balance:

1. draw control volume



2. Does it satisfy assumptions for an engineering balance?
   - discrete outlet ✓
   - constant density ✓

3. Balance + math:

$$\frac{dm}{dt} = \cancel{w_{in}}^{0} - w_{out}$$

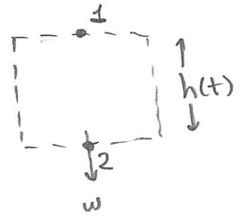
$$= -w_{out}$$

$$\frac{d}{dt}\Big(\underbrace{\rho\frac{\pi D^2}{4}}_{\text{const}} h\Big) = -\rho\frac{\pi D_{out}^2}{4} v_{out}$$

$$\frac{dh}{dt} = -\frac{D_{out}^2}{D^2} v_{out}$$ ← hmm, doesn't help much, we want to know w_{out}.

mechanical energy balance

1. Draw control volume.



2. Does it satisfy assumptions:
- single I/O ✓
- unidirectional flow @ I/O ✓
- small viscous stress @ I/O ✓
- const density ✓
- fixed C.V ? - steady ?

if $\frac{dh}{dt} \approx 0$, then we are ok. ⇒ need $D_{out} \ll D$.

* Reasonable assumption. → "Quasi-steady"

3. Balance + Math:

$$\frac{b_2 v_2^2}{2} + \frac{P_2}{\rho} + gh_2 - \frac{\cancel{b_1 v_1^2}^{0}}{2} - \frac{P_1}{\rho} - gh_1 = \frac{1}{w}\left(\cancel{W_m}^{0} - \cancel{E_v}^{0}\right)$$

(dh/dt) — no pumps / turbines

$b_2 = 1$ (turbulent) $P_1 = P_2 = P_{atm}$

$$\frac{v_2^2}{2} + g(h_2 - h_1) = 0 \Rightarrow v_2 = \sqrt{2gh}$$

$$w = \rho\frac{\pi D_{out}^2}{4} v_{out} \Rightarrow \boxed{w = \frac{\rho\pi D_{out}^2}{4}\sqrt{2gh}}$$

Evaluation: * we can calculate ω, if we measure h & D_{out} only!

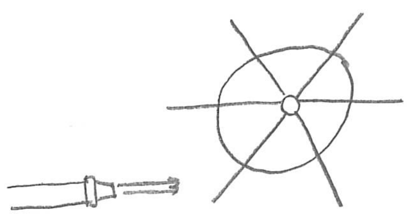
* h is going to change with time.

* Our analysis will fail if $D_{out} \approx D$

* Now we could iterate if necessary.

B. Pelton turbine Design.

* Suppose I want to design a "better" type of water wheel using a jet of water. I think using this jet will speed up my wheel and maximize power output. Optimize the design.



(aka. Pelton wheel turbine)

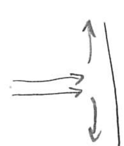
* Design:

(students do)

- Analysis:
  - Optimize Jet.
  - Number of paddles on the wheel
  - materials: light weight? Heavy?
  - Design paddles ← This will be my goal (because I also want to review integral balances!)

(I do)

- Synthesis: (1) Generate possible solutions:



flat paddles

angled paddles

there are of course others, but this makes for a good example

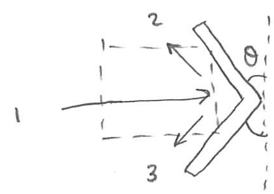
(2) Pick one & solve it:

- lets compare both. we'll solve for angled paddles, then set $\theta = 0$ for flat.

- which balance?

maybe mass, definitely momentum. Jet momentum → force on wheel.

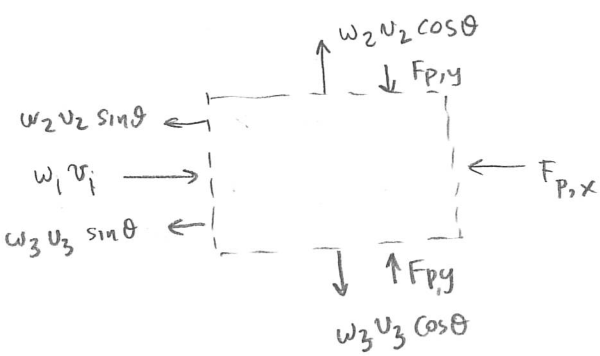
1. Draw C.V.

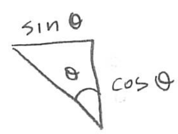


2. Does it satisfy assumptions?

- discrete I/O ✓
- unidirectional flow @ I/O ✓ approximate, but ok.
- small viscous stress @ I/O ✓
- uniform density @ I/O ✓
- fixed C.V. ✓

3. Balance + Math





momentum balance: $\dfrac{d(m\underline{v})}{dt} = \sum_i^{in} a_i w_i \underline{v}_i - \sum_i^{out} a_i w_i \underline{v}_i + \sum \underline{F}_i$

x-dir: $\dfrac{d(m v_x)}{dt} = w_1 v_1 + w_2 v_2 \sin\theta + w_3 v_3 \sin\theta - F_{p,x}$

- assume steady: $\dfrac{d(m v_x)}{dt} = 0$, what is steady state power?

- assume symmetric: $w_2 = w_3 \Rightarrow v_2 = v_3$

$$0 = w_1 v_1 + 2 w_2 v_2 \sin\theta - F_{px}$$

* by a mass balance :

$$\frac{dm}{dt} = w_1 - w_2 - w_3 \Rightarrow w_1 = w_2 + w_3 = 2 w_2$$

$\frac{dm}{dt} \to 0$

$$w_1 = \rho A_1 v_1 = \underbrace{2 \rho A_2 v_2}_{w_2}$$ ← need A_2

lets Assume w_2 & w_3 are each ½ area of A_1 : $A_2 = A_1/2$

(stream splits, but doesn't focus)

$$v_2 = \frac{A_1 v_1}{2 A_2} = \frac{A_1 v_1}{2 \frac{A_1}{2}} = v_1$$

$$0 = w_1 v_1 + w_1 v_1 \sin\theta + F_{p,x}$$

$$\boxed{F_{p,x} = w_1 v_1 (1 + \sin\theta)}$$ ← Force on wheel.

if $\theta = 0$, $F_{px} = w_1 v_1$

if $\theta = \pi/2$, $F_{px} = 2 w_1 v_1$ ← 2x the force!

- Evaluation :
  - Can we really get $\pi/2$ (full reflection) ?
  - Could we do better? what if we could focus the stream so $A_2 < A_1/2$?
  - what if it isn't symmetric? How important is that?
  - Safety, Heath, Environmental consequences?