Home work 6 - problem 3 - Bernoulli's equation

(a) Euler's equation

$$\rho\left(\frac{\partial v}{\partial t} + \underline{v}\cdot\nabla v\right) = -\nabla P$$

assume steady

$$\underline{v}\cdot\nabla\underline{v} = -\frac{1}{\rho}\nabla P$$

Take the dot product of both sides w/ $\underline{v}$.

$$\underline{v}\cdot(\underline{v}\cdot\nabla\underline{v}) = \underline{v}\cdot\left(-\frac{1}{\rho}\nabla P\right)$$

$$\underline{v}\cdot\left[\nabla\left(\frac{v^2}{2}\right) - v\times(\nabla\times v)\right] = -\underline{v}\cdot\left(\frac{1}{\rho}\nabla P\right)$$

$\longleftarrow$ density is constant

$$v\cdot\nabla\left(\frac{v^2}{2}\right) - \underbrace{\underline{v}\cdot(v\times(\nabla\times\underline{v}))}_{\underline{v}\cdot(\underline{v}\times\underline{a})=0} = -\underline{v}\cdot\nabla\left(\frac{P}{\rho}\right)$$

$$\underline{v}\cdot\nabla\left(\frac{v^2}{2}\right) + \underline{v}\cdot\nabla(P/\rho) = 0$$

$$\boxed{\underline{v}\cdot\nabla\left(\frac{v^2}{2} + \frac{P}{\rho}\right) = 0}$$

(b) This means that $\frac{v^2}{2} + \frac{P}{\rho}$ = constant along a streamline

or that $\Delta(v^2/2) + \Delta(P/\rho) = 0$ along a streamline.