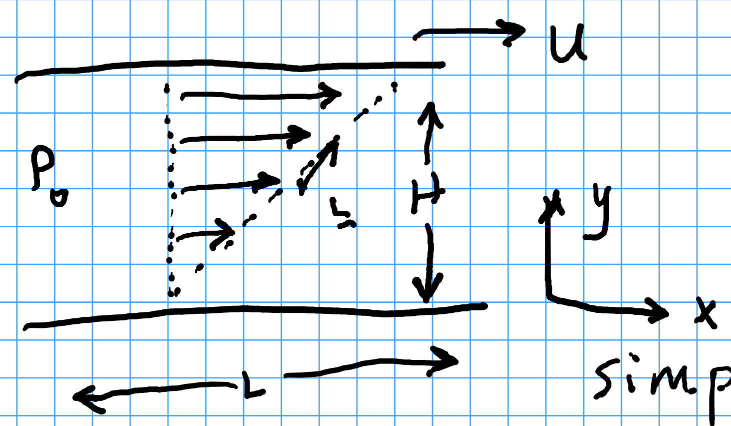
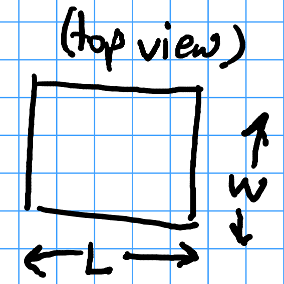
Lecture 16 – Example calculating forces



$v_x = \frac{U}{H} y$

$v_y = 0$

$P = P_0$ (const)



simple shear flow (SSF)

$\underline{F} = \underline{n} A \cdot \underline{\underline{\sigma}} \qquad \underline{\underline{\sigma}} = -\underline{\underline{\delta}} P + \underline{\underline{\tau}} \qquad \underline{\underline{\tau}} = 2\mu \underline{\underline{\Gamma}}$

(b) Calculate the local deformation: S_{local}

$S_{loc} = \underline{p} \cdot \underline{\underline{\Gamma}} \cdot \underline{p} \qquad \underline{p} = \frac{\underline{L}}{|\underline{L}|}$

$\underline{L} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ at $x = \frac{L}{2},\ y = H/2$

$|\underline{L}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$

$$S_{loc} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 0 & U/2H \\ U/2H & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 + \frac{1}{2\sqrt{2}}\frac{U}{H} & \frac{1}{2\sqrt{2}}\frac{U}{H} + 0 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{U}{4H} + \frac{U}{4H} = \frac{U}{2H}$$

(a) Calculate the rate of strain tensor

$$\underline{\underline{\Gamma}} = \frac{1}{2}\left[\underline{\nabla}\,\underline{v} + (\underline{\nabla}\,\underline{v})^T\right] = \begin{bmatrix} \partial v_x/\partial x & \frac{1}{2}\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) & \partial v_y/\partial y \end{bmatrix}$$

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$\frac{\partial v_x}{\partial x} = \frac{\partial}{\partial x}\left(\frac{U}{H} y\right) = 0 \qquad \frac{\partial v_y}{\partial x} = \frac{\partial}{\partial x}(0) = 0$

$\frac{\partial v_x}{\partial y} = \frac{\partial}{\partial y}\left(\frac{U}{H} y\right) = \frac{U}{H} \qquad \frac{\partial v_y}{\partial y} = \frac{\partial}{\partial y}(0) = 0$

$$\underline{\underline{\Gamma}} = \begin{bmatrix} 0 & U/2H \\ U/2H & 0 \end{bmatrix}$$

$\tau_w = \frac{\mu U}{H}$

s

(c) calculate the viscous stress tensor

$\underline{\underline{\tau}} = 2\mu \underline{\underline{\Gamma}}$

$$\underline{\underline{\tau}} = \begin{bmatrix} 0 & 2\mu U/2H \\ \frac{2\mu U}{2H} & 0 \end{bmatrix}$$

$$\underline{\underline{\tau}} = \begin{bmatrix} 0 & \mu U/H \\ \frac{\mu U}{H} & 0 \end{bmatrix}$$

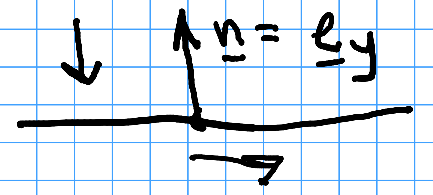
(d) Total stress tensor, $\underline{\underline{\sigma}}$

$$\underline{\underline{\sigma}} = -P\,\underline{\underline{\delta}} + \underline{\underline{\tau}} \qquad P = P_0 \qquad \underline{\underline{\delta}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= -\begin{bmatrix} P_0 & 0 \\ 0 & P_0 \end{bmatrix} + \begin{bmatrix} 0 & \mu U/H \\ \frac{\mu U}{H} & 0 \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} -P_0 & \mu U/H \\ \mu U/H & -P_0 \end{bmatrix}$$

(e) Calculate the force on the bottom plate from SSF



$$F = \underline{n} A \cdot \underline{\underline{\sigma}} \qquad A = LW$$

$$\underline{e}_y = [0 \quad 1]$$

$$\underline{F} = \underline{e}_y (LW) \cdot \underline{\underline{\sigma}}$$

$$= [0 \quad LW] \cdot \begin{bmatrix} -P_0 & \mu U/H \\ \frac{\mu U}{H} & -P_0 \end{bmatrix}$$

$$= \left[0 + \frac{\mu U}{H} LW \qquad 0 + -P_0 LW\right]$$

$$\underline{F} = \left[\underbrace{\frac{\mu U}{H} LW}_{F_x} \qquad \underbrace{-P_0 LW}_{F_y}\right]$$