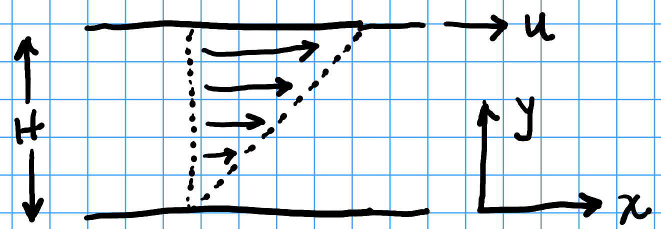
Lecture 14 - Example : Simple Shear Flow



$v_x = \frac{Uy}{H}$

$v_y = 0$

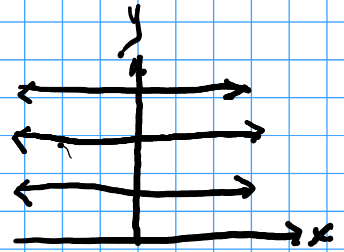
- translation → streamlines } today!
- rotation → vorticity
- deformation → shear rate $\dot{\gamma}$ (tensor)

(a) What are the streamlines? ← ψ : streamfunction

$$v_x = \frac{\partial \psi}{\partial y} \qquad v_y = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial \psi}{\partial y} = v_x = \frac{Uy}{H} \Rightarrow \psi = \frac{Uy^2}{2H} + c_1(x)$$

$$\frac{\partial \psi}{\partial x} = \frac{\partial c_1}{\partial x} = 0 \Rightarrow \boxed{\psi = \frac{1}{2}\frac{Uy^2}{H} + c_1} = \cancel{\text{const}}\ c_2$$



$\psi = \text{const}$

$$\frac{1}{2}\frac{Uy^2}{H} = c_2 - c_1$$

$$y^2 = \frac{(c_2 - c_1)2H}{U} \Rightarrow \boxed{y = \left[\frac{2H c_3}{U}\right]^{1/2}}$$

(b) What is the vorticity? ← rotation

$\underline{w} = \underline{\nabla} \times \underline{v}$ (Table 5.3) in Deen

$$= \left(\cancel{\frac{\partial v_z}{\partial y}} - \cancel{\frac{\partial v_y}{\partial z}}\right) e_x + \left(\cancel{\frac{\partial v_x}{\partial z}} - \cancel{\frac{\partial v_z}{\partial x}}\right) \underline{e}_y$$

$$\left(\cancel{\frac{\partial v_y}{\partial x}} - \frac{\partial v_x}{\partial y}\right) \underline{e}_z$$

$$= -\frac{\partial}{\partial y}\left(\frac{Uy}{H}\right) \underline{e}_z = \boxed{-\frac{U}{H}\ \underline{e}_z}$$