Homework 8

Ch En 374 – Fluid Mechanics

Due date: 8 Nov. 2019

Survey Question

Please report how long it took you to complete this assignment (in hours) in the "Notes" section when you turn your assignment in on Learning Suite.

Practice Problems

1. [Exam Review Questions]. Try and answer the following True/False questions from memory without looking at your book. Then use your book or a friend and fix your answers.

   (a) Write the general expression for the force on a surface due to a stress in a fluid.

   (b) What is the incompressible continuity equation? What does it mean physically?

   (c) Write down the x-component of the Navier-Stokes equation in Cartesian coordinates. What terms remain if I assume that the flow is unidirectional in x but *unsteady*?

   (d) T/F. $\boldsymbol{\tau}$ is the rate-of-strain tensor.

   (e) T/F. The no-slip condition means that the fluid next to a solid surface moves at the same velocity as that surface.

   (f) T/F. The Navier-Stokes equation is analogous to Newton's 2nd law.

   (g) T/F. A Lagrangian reference frame is one that moves relative to the fluid.

   (h) T/F. Vorticity is a mathematical expression of the rotation of the flow.

   (i) T/F. $\boldsymbol{\tau}_{zz}$ is a normal stress.

   (j) T/F. The Cauchy momentum equation applies only to Newtonian Fluids.

2. [Lecture 23 – Turbulent Flow]. In class we derived the velocity profiles for laminar pipe flow,

$$v_z(r) = 2U\left[1 - \left(\frac{r}{R}\right)^2\right]$$

and for turbulent flow,

$$\langle v_z(r)\rangle = U\frac{2.5\ln[R^+(1 - r/R)] + 5.5}{2.5\ln(R^+) + 1.75}$$

where R is the pipe radius, U is the mean velocity and R^+ is a dimensionless function of the wall shear stress τ_w. Because the latter expression is sometimes cumbersome to use, an alternative, empirically-determined average velocity profile for turbulent flow

$$\langle v_z(r)\rangle = U\frac{60}{49}\left(1 - \frac{r}{R}\right)^{1/7}$$

is sometimes used. Plot all three velocity profiles (v_z/U vs. r/R) using your favorite numerical tool assuming $R^+ = 1000$. Sometimes in engineering it is convenient to assume "plug flow", meaning a flat (i.e. constant velocity) profile. Based on your plot, which flow regime (laminar or turbulent) is closer to plug flow? Additionally, comment on why an average velocity profile is needed for turbulent flow, but is not for laminar flow.

3. [Lecture 24 – CFD]. Laplace's equation

$$\nabla^2 P = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0$$

is like Poisson's equation, but the right hand side is zero. (We have seen Laplace's equation before, when we talked about irrotational flow.) Both are partial differential equations that can be solved using finite differences.

(a) Use the finite difference formulas we wrote in class and discretize Laplace's equation on a grid. Then solve for P_{ij} in terms of the P's at other grid points, as well as Δx and Δy.

(b) Isolating P_{ij} is the first step in writing a method to solve Laplace's or Poisson's equation, and this method has been used in the CFD code I provided for you in class. Print out the relevant portion of the CFD program, and highlight the lines where this part is coded. Turn in your highlighted page.

Challenge Problems

4. As we saw in problem 2 above, in addition the function we described in class, the average velocity profile for turbulent flow in a pipe can also be described by

$$\langle v_z(r) \rangle = U \frac{60}{49} \left(1 - \frac{r}{R}\right)^{1/7}$$

where U is the average velocity and R is the pipe radius. We would like to get an equation for the friction factor using this velocity profile.

(a) Using derivatives of the 1/7-power expression, find the shear stress at the wall, τ_w. Why can't you use this τ_w to find the friction factor? (*Note that this is the same thing that happens for the logarithmic velocity profile we derived in class.*)

(b) Because the previous approach doesn't work, we must instead use a different expression of the 1/7-power velocity profile,

$$\langle v_z(r) \rangle = 8.75 \left(\frac{\tau_w}{\rho}\right)^{1/2} \left[R^+ \left(1 - \frac{r}{R}\right)\right]^{1/7}$$

where $R^+ = R\sqrt{\rho \tau_w}/\mu$. Use this equation to find the friction factor and compare it to the Blasius correlation for smooth pipes [Eq. (2.2-9) in your book].

5. Using the Jupyter notebook I provided in class, calculate the drag coefficient of a square cylinder for Re = 10, 20, 50, 100, 200, and 500. For the calculations, use `Lx` = 5.5, `Ly` = 1, `Np` = 5000, `box_width` = 0.2, `box_height` = 0.2, `box_x` = 2, `box_y` = 0.5 and the Reynolds number-dependent model parameters in the table to the right. Using a separate Python or Excel file, make a log-log plot of the drag coefficient versus the Reynolds number. Plot Eq. (3.2-9) from Deen (the drag coefficient of a cylinder) alongside the drag coefficients that you calculate. *(Hint: These simulations can take several minutes each, so plan accordingly.)*

Re	`dt`	`Nt`	`Nx`	`Ny`
10	1×10^{-4}	10001	353	65
20	1×10^{-4}	10001	353	65
50	5×10^{-5}	10001	353	65
100	5×10^{-5}	20001	353	65
200	5×10^{-5}	20001	353	65
500	5×10^{-5}	20001	1079	197