Homework 7

Ch En 374 – Fluid Mechanics

Due date: 1 Nov. 2019

Survey Question

Please report how long it took you to complete this assignment (in hours) in the "Notes" section when you turn your assignment in on Learning Suite.

Practice Problems

1. [Lecture 20 – Creeping and Inviscid Flow]. Write the correct simplified version of the Navier-Stokes equation that applies in the following situations:

   (a) The outer region around an airplane wing in high speed air flow where the vorticity is zero.

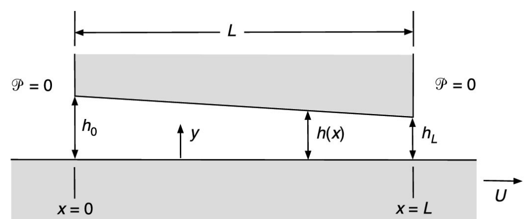
   (b) The inner region around an airplane wing in high speed air flow where the vorticity is zero.

   (c) High Reynolds number ocean currents with local rotation.

   (d) E. coli swimming in culture broth.

   A couple ideas in this problem are related to Lecture 22 on boundary layer theory.

2. [Lecture 21 – Drag Force Calculations]. In the slider bearing problem from HW 5, we had two nearly parallel surfaces as shown in the image on the right. Recall that the bottom surface of length L and width W moves to the right with speed U and the top, slightly inclined surface at height



$$h(x) = \frac{h_L - h_0}{L}x + h_0$$

   is stationary. The normal vector to the top surface is given by

$$\boldsymbol{n} = \frac{1}{g}\frac{dh}{dx}\boldsymbol{e}_x - \frac{1}{g}\boldsymbol{e}_y, \quad g(x) = \left[1 + \left(\frac{dh}{dx}\right)^2\right]^{1/2}$$

   We would like to know the x-component of the force on the top surface. Simplify the total force equation

$$\boldsymbol{F} = \int \boldsymbol{n} \cdot \boldsymbol{\sigma} dA$$

   to find the x-component $\boldsymbol{F} \cdot \boldsymbol{e}_x$ in terms of P and the Cartesian components of $\boldsymbol{\tau}$ (e.g. τ_{xx}, τ_{xy}, etc). The differential area is $dA = Wgdx$. (*Hint: Your answer will still be in the form of an integral over dx. Do not plug in velocities and try and solve for the force.*)

3. [Lecture 22 – Boundary Layer Theory]. In this problem we are going to use Python to plot the velocity profile for flow over a flat plate. As we learned in class, the velocity profile has two regions. When $y < \delta$ the velocity is given by the Von Kármán–Pohlhausen (VKP) equation,

$$v_x(x, y) = U\left[\frac{3}{2}\left(\frac{y}{\delta(x)}\right) - \frac{1}{2}\left(\frac{y}{\delta(x)}\right)^3\right]$$

where

$$\delta(x) = \sqrt{\frac{840}{39}\frac{\nu x}{U}}.$$

When $y > \delta$ the velocity is given by

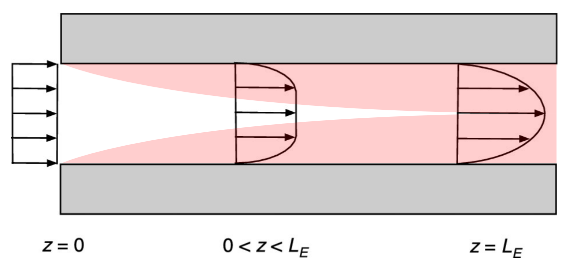
$$v_x(x, y) = U$$

Make a vector plot of this velocity profile for $x \in [0, L]$ and $y \in [0, 3\delta(L)]$ where $U = 1$ m/s, $L = 2$ m and $\nu = 10^{-6}$ m^2/s. In addition, plot the boundary layer height, $\delta(x)$ overlaid on top of the velocity field.

Challenge Problems

4. In any long pipe or other conduit of constant cross-section, the velocity profile eventually becomes fully developed. The distance from the inlet at which this occurs is the entrance length L_E. Within the entrance region part of the fluid accelerates and part decelerates, depending on the initial velocity profile. If there is plug flow at the inlet, as in the figure below, the fluid at the center speeds up and that near the wall slows down, until the final parabolic profile is achieved. Accordingly, the inertial terms in the Navier-Stokes equations are not zero, as they are in the fully developed region.

   Despite these complications, we can use boundary layer theory to get an estimate of the entrance length. The basic idea is to assume that each opposing wall has a boundary layer that eventually meets in the middle, fully enveloping the pipe.

   (a) Use the VKP equations to generate an expression for the entrance length L_E in laminar flow in terms of the average velocity U, the viscosity μ, the pipe diameter D, and the density ρ.

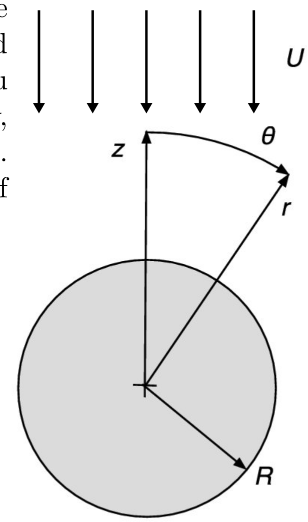
   (b) Use the VKP equations to calculate the drag force on the walls F_D of the pipe along the entrance length in terms of μ, U, D, L_E and ρ.

   (c) Calculate an entrance length friction factor using

   $$f = \frac{2F_D/A_{\parallel}}{\rho U^2}.$$

   How does this friction factor compare to the one obtained from fully developed laminar pipe flow? How does it compare to the formula for the coefficient of friction (C_f) for a flat plate in equations 3.2-15 and 9.3-18 in your textbook?

5. You are drinking a Coke in the CougarEat, and you notice a bubble rising in your drink. It seems to be moving at a constant velocity, and you'd like to get a prediction of how fast the bubble is moving. You reason that because the bubble is small and is not moving very quickly, the Reynolds number might be small enough to assume creeping flow. For creeping flow, the velocity and pressure fields around a bubble of radius R moving at velocity U are given by,



$$v_r(r,\theta) = -U\cos\theta\left(1-\frac{R}{r}\right)$$
$$v_\theta(r,\theta) = U\sin\theta\left(1-\frac{R}{2r}\right)$$
$$\mathcal{P}(r,\theta) = \frac{\mu U R}{r^2}\cos\theta$$

in the (spherical) coordinate system shown on the right. Note that the velocity field is axisymmetric.

(a) Using the expressions for the velocity and pressure and the simplified drag force formula,

$$F_D = 2\pi R^2 \int_0^\pi \left[-\mathcal{P}(R,\theta)\cos\theta + \tau_{rr}(R,\theta)\cos\theta - \tau_{r\theta}(R,\theta)\sin\theta\right]\sin\theta d\theta$$

calculate the drag coefficient for the bubble.

(b) Derive an expression for the terminal velocity of the bubble and use your expression to calculate the velocity of a air bubble in Coke. Make your best guess as to property values and sizes. Calculate the Reynolds number. Were you justified in assuming creeping flow? (You will get credit if you make reasonable assumptions.)