Homework 3

Ch En 374 – Fluid Mechanics

Due date: 27 Sept. 2019

Survey Question

Please report how long it took you to complete this assignment (in hours) in the "Notes" section when you turn your assignment in on Learning Suite.

Practice Problems

1. [Lecture 8 – Pipe Flow II]. Air enters a 10 m long section of a rectuangular duct of cross section 15 cm × 20 cm made of commercial steel at 1 atm and 35°C at an average velocity of 7 m/s. Disregarding entrance effects, determine the fan power needed to overcome the pressure losses in this section of the duct. (At 1 atm and 35°C, $\rho_{\mathrm{air}} = 1.145\,\mathrm{kg/m^3}$ and $\mu_{\mathrm{air}} = 1.895\times10^{-5}\,\mathrm{kg/ms}$.) *Hint: Power can be found by taking the flow rate times the absolute value of the pressure drop.*

2. [Lecture 9 – Pipe Flow III: Non-Newtonian Fluids]. You have a polymer solution with a density $\rho = 60$ lbm/ft^3 that you suspect obeys a power-law constitutive relationship ($\tau = ms^n$). To test this, you obtain the following data from flow in two horizontal cylindrical pipes.

Pipe Diameter (in)	Flowrate (ft^3/min)	Pressure Loss (psi)	Pipe Length (ft)
1.0	0.075	4.9	280
1.0	0.20	10.0	280
1.0	0.41	17.0	280
1.0	0.68	24.6	280
1.0	1.00	32.5	280
1.0	1.36	40.9	280
1.5	5.75	32.2	280
1.5	6.79	36.8	280
1.5	7.36	41.3	280
1.5	7.71	46.5	280
1.5	8.11	52.7	280
1.5	8.33	59.0	280

Using some detailed calculations that we will do later in the course, it is possible to show that for laminar flow

$$\ln \tau_w = n \ln\left(\frac{U}{R}\right) + \ln m + n \ln\left(\frac{3n+1}{n}\right) \tag{1}$$

where

$$\tau_w = R\,|\Delta\mathcal{P}|\,/(2L) \tag{2}$$

is the wall shear stress, R is the pipe radius, $\mathcal{P}$ is the dynamic pressure, L is the pipe length and $U = Q/(\pi R^2)$ is the average velocity. Use these expressions to fit the data and obtain the value of n and m (be careful about units). Is the fluid Newtonian, shear-thinning or shear-thickening? *Hint: Fit only the data in the linear regime at low Reynolds number. After you have n and m, you should be able to verify that the non-linear regime occurs when $Re > 2000$.*

3. [Lecture 10 – Drag]. Consider a refrigeration truck traveling at 70 mph. The refrigerated compartment of the truck can be considered to be a 9-ft-wide, 8-ft-high, and 20-ft-long rectangular box. Determine the drag force acting on the truck assuming a drag coefficient of $C_D = 0.96$. In addition, plot the power consumption due to drag as a function of speed from 55 mph to 80 mph.

Challenge Problems

4. (a) This function

$$C_D = \frac{24}{\mathrm{Re}} + \frac{2.6\left(\frac{\mathrm{Re}}{5.0}\right)}{1+\left(\frac{\mathrm{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\mathrm{Re}}{2.63\times10^5}\right)^{-7.94}}{1+\left(\frac{\mathrm{Re}}{2.63\times10^5}\right)^{-8.00}} + \frac{0.25\left(\frac{\mathrm{Re}}{10^6}\right)}{1+\left(\frac{\mathrm{Re}}{10^6}\right)} \tag{3}$$

from Prof. Faith Morrison at Michigan Tech. is a pretty good empircal fit for the drag coefficient of a sphere over the entire ragne of Reynolds numbers. Plot this function for $\mathrm{Re} = [10^{-2}, 10^8]$ and label the relevant regimes of behavior. Explain what pressure drag, friction drag and boundary layers have to do with each of these regimes.

(b) Poiseuille's law

$$f = \frac{16}{\mathrm{Re}} \tag{4}$$

and the Colebrook–White equation

$$\frac{1}{\sqrt{f}} = -4\log_{10}\left(\frac{1.26}{\mathrm{Re}\sqrt{f}} + \frac{k}{3.7D}\right) \tag{5}$$

give the friction factor for a circular pipe with roughness k and diameter D. Plot the friction factor for $\mathrm{Re} = [10^2, 10^7]$ and $k/D = \{0, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}\}$ and label the relevant regimes of behavior. Explain what stresses are dominant in the various regimes.

5. World-class downhill ski racers reach speeds up to 150 km/h. The objective of this problem is to estimate the forces on a skier at that speed. For simplicity, the skier will be modeled as two cylinders (legs) attached to a sphere (the rest of the body including trunk, arms and head). Some very approximate dimensions are $D = 14$ cm and $L = 70$ cm for each leg and $D = 70$ cm for the rest of the body. For air at 0 °C, $\rho = 1.29\,\mathrm{kg/m^3}$ and $\nu = 1.32\times10^{-5}\mathrm{m^2/s}$

   (a) Calculate the air drag on the skier.

   (b) Also resisting the motion is the force of the skis, which is mostly due to friction where they contact the snow. Suppose that a skier with a mass of 90 kg is moving at a constant speed down a 45° slope. What fraction of the total resistance is due to drag?

6. How much power (in horsepower) is needed to pump the polymer solution from problem 2 from one tank up a vertical rise of 130 feet to another tank through 500 feet of 1.5-in diameter copper pipe at 8.33 $\mathrm{ft^3/min}$?