Properties of Water

$$\rho = 10^3\,\mathrm{kg/m^3}$$
$$\mu = 10^{-3}\,\mathrm{Pa\,s}$$
$$\gamma = 72.9\,\mathrm{mN/m}\,(\text{air interface})$$

Properties of Air at 27°C and 100 kPa

$$\rho = 1.18\,\mathrm{kg/m^3}$$
$$\mu = 1.86\times 10^{-5}\,\mathrm{Pa\,s}$$

Unit Conversions & Constants

$$1\,\mathrm{m} = 3.2808\,\mathrm{ft}$$
$$1\,\mathrm{kPa} = 0.1450\,\mathrm{psi}$$
$$1\,\mathrm{atm} = 101.3\,\mathrm{kPa} = 14.7\,\mathrm{psi}$$
$$1\,\mathrm{kg} = 2.2056\,\mathrm{lbm}$$
$$1\,\mathrm{slug} = 32.174\,\mathrm{lbm}$$
$$1\,\mathrm{lbf} = 4.448\,\mathrm{N} = 1\,\mathrm{slug}\cdot\mathrm{ft/s^2}$$
$$g = 9.807\,\mathrm{m/s^2} = 32.17\,\mathrm{ft/s^2}$$

Equations

$$\boldsymbol{w} = \nabla\times\boldsymbol{v}$$

$$v_x = \frac{\partial\psi}{\partial y} \qquad v_y = -\frac{\partial\psi}{\partial x}$$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = -\frac{\partial\psi}{\partial r}$$

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla f$$

$$\boldsymbol{a} = \frac{D\boldsymbol{v}}{Dt}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0$$

$$\nabla\cdot\boldsymbol{v} = 0$$

$$\frac{1}{L}\frac{dL}{dt} = \boldsymbol{p}\cdot\boldsymbol{\Gamma}\cdot\boldsymbol{p}$$

$$\boldsymbol{\Gamma} = \frac{1}{2}\left[\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T\right]$$

$$\boldsymbol{F} = A\boldsymbol{n}\cdot\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} = -\boldsymbol{\delta}P + \boldsymbol{\tau}$$

$$\boldsymbol{F} = \int_S \boldsymbol{n}\cdot\boldsymbol{\sigma}dS$$

$$\boldsymbol{F}_D = -\int_S \boldsymbol{e}_z\cdot\boldsymbol{n}P\,dS + \int_S \boldsymbol{n}\cdot\boldsymbol{\tau}\cdot\boldsymbol{e}_z\,dS$$

$$\boldsymbol{\tau} = 2\mu\Gamma$$

$$\boldsymbol{\tau} = \mu\left[\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T\right]$$

$$f = \frac{2\tau_w}{\rho U^2}$$

$$C_D = \frac{2F_D/A_\perp}{\rho U^2}$$

$$U = \frac{1}{A}\int v_z\,dA$$

$$\dot{m} = \rho UA = \rho\boldsymbol{v}\cdot(-\boldsymbol{n}A)$$

$$\dot{\boldsymbol{p}} = \rho\boldsymbol{v}\boldsymbol{v}\cdot(-\boldsymbol{n}A)$$

$$\tau_{rz} = m\left(\frac{\partial v_z}{\partial r}^n\right)$$

$$\mathrm{Re}_{\mathrm{PL}} = \frac{8\rho U^{2-n}D^n}{m\left[\frac{2(3n+1)}{n}\right]^n}$$

$$\rho\frac{D\boldsymbol{v}}{Dt} = \rho\boldsymbol{g} + \nabla\cdot\boldsymbol{\sigma}$$

$$\rho\frac{D\boldsymbol{v}}{Dt} = \rho\boldsymbol{g} - \nabla P + \mu\nabla^2\boldsymbol{v}$$

$$\nabla\mathcal{P} = \nabla P - \rho\boldsymbol{g}$$

$$0 = -\nabla\mathcal{P} + \mu\nabla^2\boldsymbol{v}$$

$$\rho\frac{D\boldsymbol{v}}{Dt} = -\nabla\mathcal{P}$$

$$\boldsymbol{v} = \nabla\phi$$

$$\nabla^2\phi = 0$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial \mathcal{P}}{\partial x} + \nu \frac{\partial^2 v_x}{\partial y^2}$$

$$v_x(x,y) = U\left[\frac{3}{2}\left(\frac{y}{\delta(x)}\right) - \frac{1}{2}\left(\frac{y}{\delta(x)}\right)^3\right]$$

$$\delta(x) = \sqrt{\frac{840}{39}\frac{\nu x}{U}}$$

$$\rho \frac{D\langle \boldsymbol{v}\rangle}{Dt} = -\nabla\langle P\rangle + \mu\nabla^2\langle \boldsymbol{v}\rangle + \nabla\cdot\boldsymbol{\tau}^*$$

$$\boldsymbol{\tau}^* = -\rho\langle \boldsymbol{u}\boldsymbol{u}\rangle$$

$$\tau^*_{xy} = \rho\epsilon\frac{\partial\langle v_x\rangle}{\partial y}$$

$$\nabla^2 P = -\nabla\cdot(\boldsymbol{v}\cdot\nabla\boldsymbol{v})$$

$$\frac{df}{dx} = \frac{f_{i+1} - f_i}{\Delta x}$$

$$\frac{d^2 f}{dx^2} = \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2}$$

Expanded Vector Equations

Cartesian Coordinates

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\rho\left[\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right] = -\frac{\partial\mathcal{P}}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right] = -\frac{\partial\mathcal{P}}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial\mathcal{P}}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\Gamma_{xx} = \frac{\partial v_x}{\partial x}, \quad \Gamma_{yy} = \frac{\partial v_y}{\partial y}, \quad \Gamma_{zz} = \frac{\partial v_z}{\partial z}$$

$$\Gamma_{xy} = \Gamma_{yx} = \frac{1}{2}\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$

$$\Gamma_{yz} = \Gamma_{zy} = \frac{1}{2}\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$

$$\Gamma_{zx} = \Gamma_{xz} = \frac{1}{2}\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$$

$$\tau_{xx} = 2\mu\frac{\partial v_x}{\partial x}, \quad \tau_{yy} = 2\mu\frac{\partial v_y}{\partial y}, \quad \tau_{zz} = 2\mu\frac{\partial v_z}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$

$$\tau_{yz} = \tau_{zy} = \mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$

$$\tau_{zx} = \tau_{xz} = \mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$$

Cylindrical Coordinates

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right] = -\frac{\partial\mathcal{P}}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right] = -\frac{1}{r}\frac{\partial\mathcal{P}}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

$$\rho\left[\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial\mathcal{P}}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\Gamma_{rr} = \frac{\partial v_r}{\partial r},\ \Gamma_{\theta\theta} = \left[\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right],\ \Gamma_{zz} = \frac{\partial v_z}{\partial z}$$

$$\Gamma_{r\theta} = \Gamma_{\theta r} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\Gamma_{\theta z} = \Gamma_{z\theta} = \frac{1}{2}\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$$

$$\Gamma_{zr} = \Gamma_{rz} = \frac{1}{2}\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

$$\tau_{rr} = 2\mu\frac{\partial v_r}{\partial r},\ \tau_{\theta\theta} = 2\mu\left[\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right],\ \tau_{zz} = 2\mu\frac{\partial v_z}{\partial z}$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$$

$$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

Spherical Coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho v_\phi) = 0$$

$$\rho\left[\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta^2}{r} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right] =$$
$$-\frac{\partial \mathcal{P}}{\partial r} + \mu\left[\nabla^2 v_r - \frac{2}{r^2}v_r - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} - \frac{2}{r^2}v_\theta\cot\theta - \frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial\phi}\right]$$

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right] =$$
$$-\frac{1}{r}\frac{\partial\mathcal{P}}{\partial\theta} + \mu\left[\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial\phi}\right]$$

$$\rho\left[\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial\theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi\cot\theta}{r}\right] =$$
$$-\frac{1}{r\sin\theta}\frac{\partial\mathcal{P}}{\partial\phi} + \mu\left[\nabla^2 v_\phi - \frac{v_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)$$

$$\Gamma_{rr} = \frac{\partial v_r}{\partial r}$$

$$\Gamma_{\theta\theta} = \left[\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right]$$

$$\Gamma_{\phi\phi} = \left[\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right]$$

$$\Gamma_{r\theta} = \Gamma_{\theta r} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial\theta}\right]$$

$$\Gamma_{\theta\phi} = \Gamma_{\phi\theta} = \frac{1}{2}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$\Gamma_{\phi r} = \Gamma_{r\phi} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$

$$\tau_{rr} = 2\mu\frac{\partial v_r}{\partial r}$$

$$\tau_{\theta\theta} = 2\mu\left[\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right]$$

$$\tau_{\phi\phi} = 2\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right]$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial\theta}\right]$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = \mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right]$$

$$\tau_{\phi r} = \tau_{r\phi} = \mu\left[\frac{1}{r\sin\theta}\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right]$$