Homework 9

Ch En 263 – Numerical Tools

Due: 18 Mar. 2024

Instructions

- Complete the problems below and submit the following files to Learning Suite:
  - Handwritten portion: scan each page (or take a picture) and combine them into a single pdf named: `LastName_FirstName_HW9.pdf`
  - Excel portion: submit a workbook named `LastName_FirstName_HW9.xlsx` where each worksheet tab is named "Problem_1", "Problem_2", etc.
  - Python portion: submit a separate file for each problem named `LastName_FirstName_HW9_ProblemXX.py` where XX is the problem number.

Problems

1. Use Excel's Solver to solve the Colebrook equation for the friction factor, f

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

   when $\epsilon = 0.025$ mm, $D = 50$ mm and $\text{Re} = 25000$. *Hint: From physical intuition (in a class you will take in the future!) one knows that f will be greater than 0 and smaller than 1.*

2. The equations below come from a kind of vapor-liquid equilibrium problem that you will solve in your Thermodynamics class.

$$x_1 10^{A_1 - B_1/(T+C_1)} = p_1$$
$$(1 - x_1)10^{A_2 - B_2/(T+C_2)} = p_2$$

   In these problems, one has a mixture of two chemicals (e.g. ethylbenzene and toluene) that are partially liquid and partially vapor. x_1 is the mole-fraction of species 1 in the liquid, $x_2 = 1 - x_1$ is the mole-fraction of species 2 in the liquid, T is the temperature in °C, p_1 is the partial pressure of species 1 in the vapor, p_2 is the partial pressure of species 2 in the vapor and A_1, B_1, C_1, A_2, B_2, C_2 are Antoine Coefficients for species 1 and 2 respectively.

   Given that $p_1 = 250$ mmHg, $p_2 = 343$ mmHg, $A_1 = 6.95719$, $B_1 = 1424.255$, $C_1 = 213.21$, $A_2 = 6.95464$, $B_2 = 1344.8$, $C_2 = 219.48$, use Scipy's `minimize` function to find the mole-fraction x_1 and the temperature T in °C.

   Hint: Don't worry about messing with the units for this problem; they are kind of screwy in these "flash" problems because of the $10^{A-B/(T+C)}$ part. If you just plug in the numbers I've given you, they will work out.

3. We have a parallel pipe network with three pipes. We want to find the flow rates Q_i through each pipe. The flow rates are written in terms of the friction factors f_i for each pipe, which gives six equations,

$$Q_0 + Q_1 + Q_2 = Q_{tot} \tag{1}$$

$$\frac{f_0 L_0 \rho}{2D_0}\left(\frac{4Q_0}{\pi D_0^2}\right)^2 = \frac{f_1 L_1 \rho}{2D_1}\left(\frac{4Q_1}{\pi D_1^2}\right)^2 \tag{2}$$

$$\frac{f_0 L_0 \rho}{2D_0}\left(\frac{4Q_0}{\pi D_0^2}\right)^2 = \frac{f_2 L_2 \rho}{2D_2}\left(\frac{4Q_2}{\pi D_2^2}\right)^2 \tag{3}$$

$$\frac{1}{\sqrt{f_0}} = -2\log_{10}\left(\frac{\epsilon_0}{3.7D_0} + \frac{2.51\mu\pi D_0}{4\rho Q_0\sqrt{f_0}}\right) \tag{4}$$

$$\frac{1}{\sqrt{f_1}} = -2\log_{10}\left(\frac{\epsilon_1}{3.7D_1} + \frac{2.51\mu\pi D_1}{4\rho Q_1\sqrt{f_1}}\right) \tag{5}$$

$$\frac{1}{\sqrt{f_2}} = -2\log_{10}\left(\frac{\epsilon_2}{3.7D_2} + \frac{2.51\mu\pi D_2}{4\rho Q_2\sqrt{f_2}}\right) \tag{6}$$

and six unknowns, Q_0, Q_1, Q_2, f_0, f_1, f_2. The following quantities are given (in SI units):

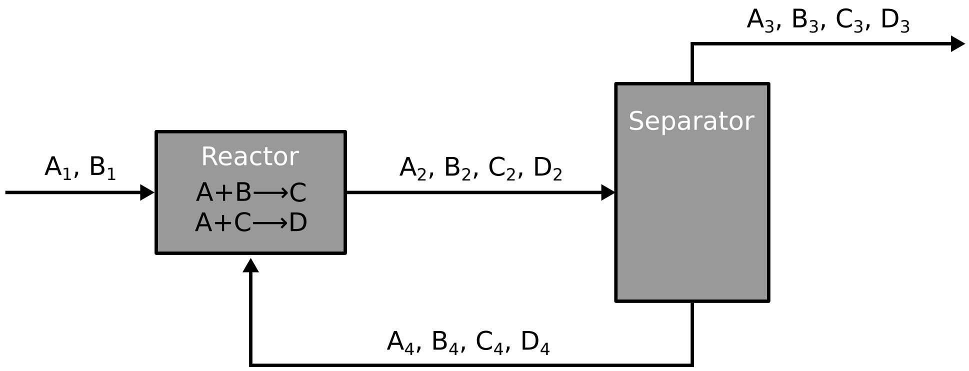
$Q_{tot} = 0.01333$ m^3/s	$\mu = 1.002 \times 10^{-3}$ kg/(m·s)	$\rho = 998$ kg/m^3
$L_0 = 100$ m	$L_1 = 150$ m	$L_2 = 80$ m
$D_0 = 0.05$ m	$D_1 = 0.045$ m	$D_2 = 0.04$ m
$\epsilon_0 = 0.00024$ m	$\epsilon_1 = 0.00012$ m	$\epsilon_2 = 0.0002$ m

Find the flow rates Q_0, Q_1 and Q_2. *Hint: It is a good idea to make your guesses consistent. That is, make sure your guesses for Q_0, Q_1, and Q_2 add up to the given Q_{tot}. Reasonable guesses for f_0, f_1, f_2 are 0.01.*

Notes on the physical interpretation of the equations (if you are interested):

- Eq. 1 says the total flow rate is the sum of the flow rates through each pipe.
- Eq. 2 says that the pressure drop through pipe 0 is the same as that through pipe 1, since pipes 0 and 1 are connected at their ends.
- Eq. 3 says that the pressure drop through pipe 1 is the same as that through pipe 2, since pipes 1 and 2 are connected at their ends.
- Eq. 4–6 relate the friction factor in each pipe to its flow rate and pipe properties.
- In the Eq. 2 and Eq 3, $4Q_i/(\pi D_i^2)$ is the velocity in the pipe. We substituted $v_i = 4Q_i/(\pi D_i^2)$ for convenience, but it makes the equations a bit harder to read. We could also define local terms like $v_0 = 4Q_0/(\pi D_0^2)$, and then use v_0 in the equations above instead of $4Q_0/(\pi D_0^2)$.
- In Eq. 4–6, $\text{Re} = \dfrac{4\rho Q_i}{\mu\pi D_i}$. Again, we made the substitution for convenience.

4. The process shown in the figure below consists of two unit operations, a reactor and a separator.



The process takes in some flow rates A_1 and B_1 of species A and B. The reactor has a reaction

$$A + B \to C \text{ (Reaction 1)}$$

that produces an intermediate product C that then needs to be converted to the desired product D by a second reaction

$$A + C \to D \text{ (Reaction 2).}$$

The single-pass conversion of the reactor is 90% with a 30% selectivity for Reaction 2. The products of the reaction in stream '2" are fed to a separator, which produces the product stream "3" and a recycle stream "4." The specifications of the separator are a recycle of 65% of D and 85% of C. The flow B_2 is evenly split between the product and recycle streams, while 10% of A_2 is lost to the product stream. Steady state mass balances and the other specifications give the following equations:

Reactor Mass Bal.		Separator Mass Bal.	
$A_4 - A_2 - \xi_1 - \xi_2 = -A_1$	(2.1)	$A_2 - A_3 - A_4 = 0$	(2.5)
$B_4 - B_2 - \xi_1 = -B_1$	(2.2)	$B_2 - B_3 - B_4 = 0$	(2.6)
$C_4 - C_2 + \xi_1 - \xi_2 = 0$	(2.3)	$C_2 - C_3 - C_4 = 0$	(2.7)
$D_4 - D_2 + \xi_2 = 0$	(2.4)	$D_2 - D_3 - D_4 = 0$	(2.8)

Conversion/Selectivity		Separator Specs	
		$D_4 - 0.65D_2 = 0$	(2.11)
$\xi_1 + \xi_2 - 0.9A_4 = 0.9A_1$	(2.9)	$B_3 - B_4 = 0$	(2.12)
$0.3\xi_1 - 0.7\xi_2 = 0$	(2.10)	$0.1A_2 - A_3 = 0$	(2.13)
		$0.85C_2 - C_4 = 0$	(2.14)

where ξ_1 and ξ_2 are the extent of reaction for Reactions 1 and 2 respectively.

If $A_1 = 4$ mol/s and $B_1 = 3$ mol/s specify the inlet flow rates of A and B, what are the amounts of A, B, C, and D coming out of the product stream of the separator at the end of the process?