

Exam 2 Review

* Prayer / Quiz / AMA

* Exam 2 Cover Sheet

* Review Sheet

* Questions about format, general questions

I. Review Game

1. Which method is appropriate for the system of equations?

$$3x + 4y = 5$$

$$x^2 - 2y = 4$$

(a) Back Substitution

(b) Newton's method \leftarrow ☒ Nonlinear system

(c) hp. linalg. solve

(d) Gauss Elimination

2. True or False: In general, there are no direct methods for solving nonlinear systems?

True. only iterative

3. what is the norm of this vector?

$$\underline{x} = [2, -3, 1, 3, 1, -1]$$

$$\| \underline{x} \| = [2^2 + (-3)^2 + 1^2 + 3^2 + 1^2 + (-1)^2]^{1/2}$$

$$= (4 + 9 + 1 + 9 + 1 + 1)^{1/2} = \sqrt{25} = 5$$

5

4. What is the asymptotic cost of this code as $n \rightarrow \infty$?

```
for i in range(n):
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```
    for j in range(n):
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        for k in range(n):
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```
            print(i, j, k)
```

$$\text{cost} \sim \theta(n^3)$$

5. What two tools can I use to solve a system of non-linear equations via an optimization strategy?

- Excel's Solver

- Scipy.optimize.minimize()

6. Suppose I have the following system of equations:

$$3x - y = 2$$

$$-2x + 4y = 1$$

write it in standard form $\underline{A} \cdot \underline{x} = \underline{b}$

$$\underline{A} = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

7. which method is usually more reliable for solving a non-linear equation: Picard's method or Newton's method?

Newton's method,

Picard's is simple, but often fails.

8. Suppose I have a python code that defines a matrix, A & a vector b for the equation

$$Ax = b.$$

what is the syntax I can use to solve this system?

$$x = \text{np.linalg.solve}(A, b)$$

9. write the formula for Newton's method

$$x^{(k+1)} = x^{(k)} - \frac{f(x^{(k)})}{f'(x^{(k)})}$$

10. Write the equation in residual form & vector notation

$$x^2 = 3x + \sin(\pi y)$$

$$x = -\frac{1}{2} \ln y$$

$$\underline{f}(\underline{x}) = \begin{bmatrix} x^2 - 3x - \sin(\pi y) \\ x + \frac{1}{2} \ln y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (\text{or})$$

$$\underline{f}(\underline{x}) = \begin{bmatrix} x_0^2 - 3x_0 - \sin(\pi x_1) \\ x_0 + \frac{1}{2} \ln x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

11. True or False. when using optimization methods to solve nonlinear equations, you don't have to write the function in residual form, because you are minimizing the squared error.

False. Always put in residual form.

code problems

- ↓ 12. Identify the method in the given code. What algorithm is this?

• Forward elimination in Gauss Elimination

13. Debug the code that solves the equation

$$\frac{1}{\sqrt{s}} = 4 \log(\operatorname{Re} \sqrt{s}) + 0.4$$

for f when $\operatorname{Re} = 2.5 \times 10^4$.

14. Suppose I want to find the roots of

$$x^3 - 3x^2 + 5x - 1 = 0$$

via Newton's method.

$$f' = 3x^2 - 6x + 5$$

Fill in the Excel sheet to solve for the root

using a guess $x^{(0)} = 0.25$

15. Solve the system of equations in Python:

$$-6x + 4y - 2z = -20$$

$$x + 3y = -2$$

$$-2y - z = -3$$

* Solution: $(x, y, z) = (1, -1, 5)$

Back to 2nd questions:

16. which methods are appropriate for linear systems
and which are appropriate for nonlinear systems?

Gauss Elimination
linear

Picard's Method
Nonlinear

Newton's Method
Nonlinear

17. which methods are appropriate for linear systems
and which are appropriate for nonlinear systems?

np.linalg.solve.
Linear

scipy.optimize.root
Nonlinear

scipy.optimize.minimize
Non linear

Excel Solver
Non linear

18. What was the asymptotic running time of Gauss Elimination
in terms of the matrix size, n ?

$$T(n) \sim \theta(n^3)$$

19. Suppose I have an algorithm that is $\Theta(n^3)$ and it takes 10 seconds to execute when $n=100$. How long will it take when $n=200$?

$$C \cdot (100)^3 = 10 \text{ sec} \rightarrow C = \frac{10 \text{ sec}}{10^6} = 10^{-5} \text{ sec.}$$

$$C \cdot (200)^3 = 8C \cdot 10^6 = 8 \cdot 10^{-5} \text{ sec} \cdot 10^6 = \boxed{80 \text{ sec.}}$$

fast way. \rightarrow or $\left(\frac{200}{100}\right)^3 \cdot 10 \text{ sec} = 2^3 \cdot 10 \text{ sec} = 80 \text{ sec}$

20. write pseudo code that evaluates the following formula in index notation:

$$C = \sum_{i=0}^4 \sum_{j=0}^4 A_{ij} B_{ji}$$

$C = 0$

for i in range $(0, 5)$:

for j in range $(0, 5)$:

$C += A[i, j] * B[j, i]$

21. How much memory does a 100×100 matrix filled with floats consume? (1 float = 8 bytes).

$$8 \text{ bytes} \cdot (100)^2 = 8 \cdot 10^4 \text{ bytes} = 80 \text{ kilobytes}$$

22. Suppose I have the following matrix

$$\begin{bmatrix} 2 & -3 \\ \textcircled{1} & -2 \end{bmatrix}$$

What is the first ratio for Gauss Elimination?

$$\text{ratio} = -\frac{1}{2} = -\frac{a_{ik}}{a_{kk}} \quad \begin{array}{l} i=1, \text{ row we are eliminating} \\ k=0, \text{ first diagonal} \end{array}$$

23. Suppose I have the following equation.

$$f(x) = x^3 - 2x - 1$$

What is the first step in Newton's method

assuming my $x_{\text{guess}} = 1$?

$$x^{(1)} = x^{(0)} - \frac{f(x^{(0)})}{f'(x^{(0)})} \quad \leftarrow f'(x) = 3x^2 - 2$$

$$f(x^{(0)}) = f(1) = 1^3 - 2 \cdot 1 - 1 = -2$$

$$f'(x^{(0)}) = f'(1) = 3 \cdot 1^2 - 2 = 1$$

$$x^{(1)} = 1 - \frac{-2}{1} = 3$$