Class 17

More Laplace

Complex Factors

- Denominator may have complex roots
  - $s^2 + d_1s + d_0$ where $d_1^2/4 < d_0$
  - Remember quadratic formula
- Example: $s^2 + 4s + 5$

Implications of Complex Factors

- Complex roots indicate oscillatory behavior
- If the sign of the real part of the complex roots is negative, convergence is expected
  - Conversely, if the real part is positive, it will diverge
- Algebra needed to invert transforms with complex roots is messy but doable
- We don't need to invert the transform to tell whether it will converge or diverge, or whether or not it will oscillate

Practice

- Will y(t) converge or diverge? Is y(t) smooth or oscillatory?

$$Y(s) = \frac{s+2}{s(s^2+4s+13)}$$

Method 1: $s^2 + 4s + 13 = (s + \_)^2 + \_$

Method 2: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 13}}{2 \cdot 1} =$

Inverting Transforms with Complex Roots in the Denominator

- There are at least two different ways to proceed as described in your text on pp. 48-49.
  - Use of complex numbers and Euler's identity
    - $\cos(\omega t) = (e^{j\omega t} + e^{-j\omega t})/2$; $\sin(\omega t) = (e^{j\omega t} - e^{-j\omega t})/2$
  - Expansion without using complex numbers, followed by completing the square to invert the transform (preferred)
    - Example 3.4

Example

$$Y(s) = \frac{s+2}{s(s^2+4s+5)} = \frac{\alpha_1}{s} + \frac{\alpha_2 s + \alpha_3}{s^2+4s+5}$$

- Find α_1:
- To get α_2 and α_3, clear denominator and match "like" terms

$$s+2 = \alpha_1(s^2+4s+5) + s(\alpha_2 s + \alpha_3) = (\alpha_1+\alpha_2)s^2 + (4\alpha_1+\alpha_3)s + \alpha_1 5$$

- s^2 terms $\rightarrow$ $\alpha_1 + \alpha_2 = 0$, so $\alpha_2 = -2/5$
- s terms $\rightarrow$ $4\alpha_1 + \alpha_3 = 1$, so $\alpha_3 = -3/5$

$Y(s) =$

Complete the square
Put into proper form for inversion

- Wanted: $s^2 + 4s + 5 = (s+b)^2 + w^2$
- How?
  b = (coefficient in front of s term)/2 = 4/2 = 2
- Knowing b, find w
  $b^2 + w^2 = 5 = 4 + w^2$, so $w = 1$

Need to Get Form in Laplace Table

$$L\{e^{-bt}\cos(\omega t)\} = \frac{s+b}{(s+b)^2+\omega^2} \qquad L\{e^{-bt}\sin(\omega t)\} = \frac{\omega}{(s+b)^2+\omega^2}$$

$$\frac{-\frac{2}{5}s - \frac{3}{5}}{(s+2)^2+1}$$

Has both an s and a number on the top

$$\frac{-\frac{2}{5}s-\frac{3}{5}}{(s+2)^2+1} = \frac{-\frac{2}{5}(s+2)+\frac{1}{5}}{(s+2)^2+1} = -\tfrac{2}{5}\left[\frac{(s+2)}{(s+2)^2+1}\right] + \frac{1}{5}\left[\frac{1}{(s+2)^2+1}\right]$$

Finally:

$$Y(s) = \frac{2}{5s} - \tfrac{2}{5}\left[\frac{(s+2)}{(s+2)^2+1}\right] + \tfrac{1}{5}\left[\frac{1}{(s+2)^2+1}\right]$$

and inverting $y(t) =$

Analyze the Equation

$$y(t) = \frac{2}{5} - \frac{2}{5}e^{-2t}\cos t + \frac{1}{5}e^{-2t}\sin t$$

- e^{-t} terms mean that the system will converge at long time
- sin and cos terms mean permanent oscillations

Whew!!

One More Practice Problem

$$Y(s) = \frac{1}{s^2 - 4s + 13}$$

What if Roots to Denominator Are:

$$\begin{bmatrix} 2+6i \\ 2-6i \\ -1 \\ -3 \\ -2 \end{bmatrix}$$

Initial Value

$$\frac{(s+2)}{(s+3)(s+4)}$$

Final Value

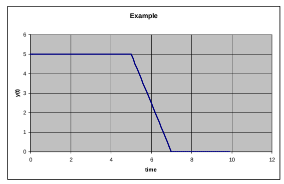
$$\frac{(s+6)}{(s+1)(s+2)}$$

Time Delay

(Fortran File)

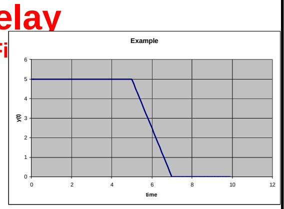
Wanted:

- Initial step to 5
- Ramp from 5 to 0 starting at t = 5 and ending at t = 7
- Final value of 0 after t = 7



Time Delay

(Fortran Fi




```
      program ft
      fun = 0.0
      S1=0.
      S2=0.
      S3=0.
      do 100 t=0.,10.,0.1
        if(t.ge.0.0)s1=1.
        if(t.ge.5.)s2=1.
        if(t.ge.7.)s3=1.
          fun=S1*5+(-5/2.)*(t-5.)*S2+5/2.*(t-7.)*S3
        print*,t,fun
100   continue
      stop
      end
```